Selective Disclosure and Scoring Bias in Contests*

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Abstract

Two ex ante identical players compete for a prize of a common, but initially unknown, value. A designer decides whether and how to disclose an informative signal of the prize's value to players and sets the scoring rule. A fully symmetric contest—with symmetrically disclosed information and a neutral scoring rule—maximizes the expected total effort. However, a tilting-and-releveling contest may maximize the expected winner's effort by distorting the contest in both dimensions to create dual asymmetry—i.e., by disclosing the signal privately to one player while biasing the scoring rule in favor of the other.

Keywords: All-pay Auction; Contest Design; Information Favoritism; Scoring Bias.

JEL Classification Codes: C72, D44, D82.

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1 Introduction

The enduring conflict between fairness and efficiency/incentives is reconciled in contest-like competitive activities that are commonplace in the modern socioeconomic landscape. These range from college admissions, sporting events, and competitive procurement (Che and Gale, 2003) to internal labor markets inside firms (Lazear and Rosen, 1981; Green and Stokey, 1983; Nalebuff and Stiglitz, 1983; Rosen, 1986). The conventional wisdom has long emphasized the importance of a level playing field as an incentive device to foster competition (see, e.g., Dixit, 1987). This insight provides a rationale for a diverse array of practices that aim to correct initial disparities among competitors. For example, in horse racing, favorite horses are often required to carry extra weight, and governments offer greater support to small and medium-sized enterprises in public procurement.

Substantial scholarly effort has been dedicated to developing various strategies to narrow the gap between asymmetric contestants who may have different levels of innate abilities. The literature typically focuses on discriminatory measures that advantage underdogs or handicap front-runners to create even races. In reality, however, contest-like competitions often take place in more complex environments. Many factors beyond the differences in players' abilities can influence their incentives. This complexity also creates greater opportunities for a designer to manipulate the rules of the competition to advance her goals.

For instance, contestants frequently face uncertainty regarding the contest's nature and the surrounding environment, such as the prize value. When competing for a promotion, employees may not fully understand the nuances of the new role—for instance, the scope of responsibilities, available resources, and implications for their career trajectory. In another context, contractors competing for government procurement may not know the true costs of fulfilling the contract. Contestants' behavior can arguably be influenced by the information available to them. This environment suggests a potential for strategic information disclosure: whether to disclose any available information and, if the designer chooses to do so, to whom.

Suppose the contest designer has discretion in two dimensions: (i) employing discriminatory measures to alter contestants' relative competitiveness and (ii) choosing to disclose or conceal prize value information among players, either symmetrically or selectively. This scenario raises numerous questions. Will a level playing field that equalizes players' relative competitiveness remain optimal? Is an equal distribution of information always optimal? How may discriminatory measures interact with the information disclosure scheme—do they substitute for or complement each other? Prior literature offers limited insights, because it typically focuses on contest design within a single dimension.

Our paper fills this gap and joins the growing literature on contest design with multiple instruments (Halac, Kartik, and Liu, 2017; Ely, Georgiadis, Khorasani, and Rayo, 2023). Focusing on the joint deployment of discriminatory measures and the information disclosure scheme, we demonstrate that the optimum could drastically depart from the conventional wisdom obtained in the context of one-dimensional contest design. In particular, we find that, when maximizing the expected *winner's* effort, the designer may optimally distort the contest in both dimensions—i.e., the information disclosure and players' relative competitiveness—to create ex post asymmetry even if players are ex ante identical.

Snapshot of the Baseline Model To highlight the contrast with the conventional wisdom, we consider two ex ante identical players who vie for a prize of a common, but initially unknown, value, which can be either high or low. The designer can acquire an informative binary signal about the true prize value. Each player's effort is converted into a score, and the higher scorer wins.

The contest rule consists of two elements. First, a disclosure scheme specifies how the signal is disclosed. It is symmetric if the signal is disclosed to or concealed from both contestants, or asymmetric if only one contestant is provided with the signal and thus awarded an information advantage. For instance, the organizer of a business pitching competition may brief preferred entrepreneurs more elaborately on the funding opportunities available to winning projects. Second, a multiplier is imposed on each player's effort to generate his score.[1] We normalize the multiplier for player 1 to one and that for player 2 to $\delta > 0$, which is called a *scoring bias*. The bias can be interpreted as a nominal judging rule, as well as measures that elevate or discount players' (perceived) output. For instance, preferred contenders competing for promotion to a higher rung on the corporate ladder may be intentionally nurtured by the incumbent CEO and board members.

We mainly consider two design objectives. The first is the usual maximization of expected *total* effort (see, e.g., Moldovanu and Sela, 2001; Moldovanu, Sela, and Shi, 2007). For instance, the government may use R&D challenges to encourage total social investment in a certain technological area (e.g., clean energy or AI). The second is the maximization of the expected *winner*'s effort (see, e.g., Moldovanu and Sela, 2006; Fu and Wu, 2022). For instance, in the competition for a corporate leadership role, the human capital gained by the winner is what will drive the value of the company.

[1] The multiplicative bid weighting rule is of both theoretical interest and practical relevance. In the "Buy American Act," the Federal Acquisition Regulation (FAR) directs the government to inflate foreign bids by an evaluation factor ranging from 6% to 50%. The evaluation factor works as an unfavorable multiplier assigned to foreign bids. See FAR Section 25 for more details.

Tilting and Releveling: Results and Extensions Absent the uncertainty in prize value, a fair contest—with $\delta = 1$—is optimal regardless of the design objective. However, with an uncertain prize value and the discretion of selective disclosure, the optimum may depart from the conventional wisdom.

A fully symmetric contest still maximizes the expected total effort, which requires symmetrically (un)informed players and a neutral scoring bias $\delta = 1$. However, when maximizing the expected winner's effort, a *tilting-and-releveling* contest could emerge in the optimum. The tilting-and-releveling contest upsets the balance of the contest in both dimensions, which creates an *ex post dual asymmetry* between the ex ante identical players. Specifically, the contest feeds the signal exclusively to one player, while releveling the playing field by biasing the scoring rule in favor of the other. The two instruments—i.e., the disclosure scheme and scoring bias—are *complementary*: Ex post asymmetry never emerges in the optimum if the designer is restricted to distorting the contest in only one dimension (Remark 1).

A player's bidding strategy depends on both his expectation of the prize value and the competition he faces. The signal allows its recipient to update his prize expectation, and thereby revise his willingness to bid—i.e., the maximum effort he may exert. Imagine that only one player is awarded the signal. Suppose that a favorable signal is realized, which elevates the recipient's prize expectation. However, he may not step up his effort, since the other player maintains his prior and his bidding strategy is independent of the realized signal. A biased scoring rule that favors the uninformed player can incentivize the informed player: The informed player has to bid more aggressively to win and is willing to do so with a favorable signal. This mechanism could enable an upward shift in the distribution of the expected winner's effort. We identify the condition under which a properly crafted tilting-and-releveling contest prevails in optimum.

Such distortion never improves the expected total effort, which is the sum of the *means* of the contestants' efforts, and thus benefits equally from the contributions of both players. In contrast, the expected winner's effort is the *modified first-order statistic* of the (random) efforts contributed by the two players, and only the winner's input matters.[2]

We extend our models to further explore the fundamentals of our analysis. First, we show that tilting-and-releveling contests may well emerge in the optimum when players are ex ante asymmetric (Section 3.3.1). The logic laid out above remains intact and governs the designer's choice of which player—the stronger or the weaker—to be an awarded information advantage or a more favorable scoring bias. Second, we take into account the designer's ability to credibly commit to her disclosure policy (Section 3.3.2). Namely, she may deviate

[2]In our context, the expected winner's effort is not necessarily the highest effort, except in the case of $\delta = 1$. We thus call the expected winner's effort a *modified* first-order statistic to reflect the nuance.

from the announced disclosure scheme when she finds it profitable, and we examine contest design with a credibility constraint. Third, we let the designer maximize the expected maximum effort, which departs from the expected winner's effort under a biased scoring rule (Section 3.3.2). In Section 4, we generalize the model to allow for a general value distribution with multiple value states. We explore information design (along with design of the scoring biases) in this setting that endogenizes the designer's information structure and the form of her signal. Our main insights remain qualitatively robust.

Related Literature Our model is a variant of the family of all-pay auctions with interdependent valuations, which include Krishna and Morgan (1997); Lizzeri and Persico (2000); Siegel (2014); Rentschler and Turocy (2016); Lu and Parreiras (2017); and Chi, Murto, and Välimäki (2019). This study is primarily linked to two strands of the literature on contest design: (i) optimal biases (as the identity-dependent differential treatment of players) and (ii) information disclosure. To the best of our knowledge, we are the first to allow the designer to choose the optimal combination of the two instruments.

The literature on optimal biases has conventionally espoused the merits of a level playing field for incentive provision—e.g., Epstein, Mealem, and Nitzan (2011); Franke, Kanzow, Leininger, and Schwartz (2013, 2014); Franke, Leininger, and Wasser (2018). A handful of recent studies—e.g., Drugov and Ryvkin (2017); Fu and Wu (2020); Barbieri and Serena (2022); Wasser and Zhang (2023); Echenique and Li (2024)—identify the contexts in which optimal biases further upset the balance of the playing field.

The literature has increasingly recognized information disclosure as a valuable addition to the toolkit for contest design. For example, Yildirim (2005); Aoyagi (2010); Ederer (2010); Goltsman and Mukherjee (2011); Halac, Kartik, and Liu (2017); Lemus and Marshall (2021); and Ely, Georgiadis, Khorasani, and Rayo (2023) examine information feedback in dynamic contests. Halac et al. (2017) and Ely et al. (2023) consider the combination of feedback scheme and prize allocation rule and focus on symmetric information disclosure. Further, the prize is allocated by outcome and cannot depend on a player's identity. In contrast, we consider a static setting and focus on the interaction between the scoring rule and disclosure scheme; we allow for selective disclosure and identity-dependent preferential treatment.

Our paper is closely related to studies of disclosing information on contestants' types, including Wärneryd (2012); Lu, Ma, and Wang (2018), Serena (2022); Zhang and Zhou (2016); Chen and Chen (2024); Melo-Ponce (2021); and Antsygina and Teteryatnikova (2023). These studies focus exclusively on disclosure schemes and portray strategic information disclosure as a device that balances competition, which aligns with the conventional wisdom of leveling the playing field. In contrast, we show that a designer may prefer information asymmetry when she controls both the disclosure scheme and scoring rule.

In the context of private-value auctions, Bergemann and Pesendorfer (2007) consider a joint design problem that allows the seller to control bidders' learning accuracy and subsequent allocation rule. They demonstrate the optimality of creating informational asymmetry together with an asymmetric follow-up design.

The rest of the paper proceeds as follows. Section 2 sets up the baseline model. Section 3 characterizes the optimal contest and presents further discussions and extensions. Section 4 deals with the case of general value distribution and endogenous information structure. Section 5 concludes. Analytical details and proofs are collected in the Appendices.

2 Baseline Model

Two risk-neutral players, indexed by $i \in \{1, 2\}$, compete for a prize of a common value $v \in \{v_H, v_L\}$, with $v_H > v_L > 0$.[3] The high value v_H is realized with a probability $\Pr(v = v_H) =: \mu \in (0, 1)$, with the low value v_L to be realized with the complementary probability. Players are initially uninformed about v, but its distribution is common knowledge. They simultaneously exert effort $x_i \geq 0$ to win the prize. One's effort incurs a constant marginal cost $c_i > 0$. For the moment, we assume that players are ex ante identical with $c_1 = c_2 = c$.[4]

Winner-selection Mechanism and Scoring Bias The contest designer imposes a scoring bias $\delta_i > 0$ on each player i's effort entry x_i, which generates his score $\delta_i x_i$. We normalize δ_1 to 1 and set $\delta_2 = \delta > 0$. We call the scoring rule with $\delta = 1$ the neutral scoring rule, which awards favoritism to neither player. The scoring rule is biased when δ deviates from 1, which favors player 2 if $\delta > 1$ and player 1 if $\delta < 1$.

A player wins if his score exceeds that of the opponent. The winner is picked randomly in the event of a tie. For given effort entries $\boldsymbol{x} := (x_1, x_2) \in \mathbb{R}^2_+$, player 1's winning probability is

$$p_1(x_1, x_2) = \begin{cases} 1, & \text{if } x_1 > \delta x_2, \\ \frac{1}{2}, & \text{if } x_1 = \delta x_2, \\ 0, & \text{if } x_1 < \delta x_2, \end{cases}$$

and player 2 wins with the complementary probability.

[3] We relax the assumption of binary value states and allow for an arbitrary discrete value distribution in Section 4.

[4] We examine the case of asymmetric players in Section 3.3.1.

Disclosure Schemes The designer conducts an investigation and obtains a verifiable noisy signal $s \in \{H, L\}$ regarding the prize value v. The signal is drawn as follows:

$$\Pr\left(s = H \,\middle|\, v = v_H\right) = \Pr\left(s = L \,\middle|\, v = v_L\right) = q, \tag{1}$$

where $q \in \left(\frac{1}{2}, 1\right]$ indicates the precision of the signal.[5] It is perfectly informative with $q = 1$ and is completely uninformative with $q = 1/2$.

The designer precommits to her disclosure scheme—i.e., how the signal will be disclosed. The disclosure scheme can formally be described by $\gamma \in \{CC, CD, DC, DD\}$, where C and D indicate "concealment" and "disclosure," respectively. With a symmetric disclosure scheme $\gamma = CC(DD)$, the realized signal s is conveyed to neither (both) of the players. With $\gamma = CD$, the designer conceals the signal from player 1 while disclosing it to player 2; $\gamma = DC$ is similarly defined.

Contest Design Prior to the contest, the designer chooses a contest scheme (γ, δ) to maximize either (i) the expected total effort, denoted by $TE(\gamma, \delta; c)$, or (ii) the expected winner's effort, denoted by $WE(\gamma, \delta; c)$.[6] The majority of the contest literature focuses on the former, which resembles revenue maximization in the auction literature. The latter, however, is relevant in a broad array of competitive activities and has attracted increasing attention in recent studies.[7]

The following notation is presented to pave the way for subsequent discussion. Let $\bar{v} := \mu v_H + (1 - \mu)v_L$ denote the ex ante expected prize value. Upon receiving a signal $s = H$, a player's expected prize value is updated to

$$\hat{v}_H(q) := \frac{\mu q v_H + (1 - \mu)(1 - q)v_L}{\mu q + (1 - \mu)(1 - q)}.$$

Similarly, the posterior upon receiving $s = L$ is

$$\hat{v}_L(q) := \frac{\mu(1 - q)v_H + (1 - \mu)q v_L}{\mu(1 - q) + (1 - \mu)q}.$$

[5] We endogenize the information structure using a Bayesian persuasion approach à la Kamenica and Gentzkow (2011) in Section 4.

[6] By maximizing the expected winner's effort, we assume the designer is committed to adopting the winning product in the context of R&D contests. If the designer lacks commitment power, she will be tempted to adopt the best product regardless of whether the contestant submitting the best product wins the contest prize. For these contests, the designer's objective is to maximize expected maximum effort. We will consider this alternative design objective in Section 3.3.3.

[7] See, e.g., Moldovanu and Sela (2006); Barbieri and Serena (2024); Fu and Wu (2022); and Wasser and Zhang, 2023).

A signal $s = H$ is realized with an ex ante probability $\hat{\mu}(q) := \mu q + (1-\mu)(1-q)$.

3 Main Results, Discussions, and Extensions

In this section, we first characterize the equilibrium of the all-pay auction under an arbitrary disclosure scheme and scoring bias. The result allows us to derive the optimal contest. We then discuss our results and present extensions to our baseline model.

3.1 Equilibrium Characterization

An all-pay auction with complete information or a discrete signal structure, in general, does not possess pure-strategy equilibria (see, e.g., Hillman and Riley, 1989; Baye, Kovenock, and De Vries, 1996; Siegel, 2009, 2010, 2014). Siegel (2014) provides a technique for constructing the unique mixed-strategy equilibrium of an all-pay auction under a neutral scoring rule, i.e., $\delta = 1$. We apply his technique in our context to characterize the equilibrium in the interim bidding stage under each (γ, δ) with an arbitrary scoring bias $\delta > 0$.

We describe by a function $b_{is}(x; \gamma, \delta)$ the equilibrium bidding strategy of a player i of type s; i.e., when receiving a signal $s \in \{H, L\}$: $b_{is}(0; \gamma, \delta, q)$ gives the *probability* that player i chooses zero effort—i.e., $x = 0$—and stays inactive, while $b_{is}(x; \gamma, \delta, q)$ provides the *probability density* of exerting an effort $x > 0$. We omit the subscript s if player $i \in \{1, 2\}$ is not granted access to the signal, so his equilibrium bidding strategy is given by $b_i(x; \gamma, \delta)$.

It can be shown that in equilibrium, a player's bidding strategy upon observing s (or nothing) consists of a probability of bidding 0 and at most two adjacent uniform bidding intervals. Specifically, the equilibrium bidding function can be represented as follows.

$$b_{is}(x;\gamma,\delta) = \begin{cases} d_0^{is}(\gamma,\delta), & \text{if } x = 0, \\ d_a^{is}(\gamma,\delta), & \text{if } 0 < (\delta \mathbb{1}_{\{i=1\}} + \mathbb{1}_{\{i=2\}})\ell_a(\gamma,\delta), \\ d_b^{is}(\gamma,\delta), & \text{if } (\delta \mathbb{1}_{\{i=1\}} + \mathbb{1}_{\{i=2\}})\ell_a(\gamma,\delta) < x \leq (\delta \mathbb{1}_{\{i=1\}} + \mathbb{1}_{\{i=2\}})\ell_b(\gamma,\delta), \\ 0, & \text{otherwise.} \end{cases}$$

Table 1 shows the values of $d_0^{is}(\gamma,\delta)$, $d_a^{is}(\gamma,\delta)$, $d_b^{is}(\gamma,\delta)$, $\ell_a(\gamma,\delta)$, and $\ell_b(\gamma,\delta)$ for the cases of $\gamma = DD$ and $\gamma = DC$. The equilibrium for the $\gamma = CC$ case can be obtained by replacing $\hat{v}_s(q)$ with $\bar{v} \equiv \mu v_H + (1-\mu)v_L$ in the case with $\gamma = DD$, and the case with $\gamma = CD$ is the mirror image of that with $\gamma = DC$.

We discuss the properties of the equilibrium in Section 3.2 after presenting the optimal contest. The equilibrium characterization enables the calculation of expected total effort

is	$d_0^{is}(\gamma,\delta)$	$d_a^{is}(\gamma,\delta)$	$d_b^{is}(\gamma,\delta)$	$\ell_a(\gamma,\delta)$	$\ell_b(\gamma,\delta)$
		$\gamma = DD,\ \delta < 1$			
$1s,\ s \in \{H,L\}$	0	$\frac{c}{\delta \hat{v}_s(q)}$	0	$\frac{\hat{v}_s(q)}{c}$	$\frac{\hat{v}_s(q)}{c}$
$2s,\ s \in \{H,L\}$	$1-\delta$	$\frac{\delta c}{\hat{v}_s(q)}$	0		
		$\gamma = DD,\ \delta \geq 1$			
$1s,\ s \in \{H,L\}$	$1-\frac{1}{\delta}$	$\frac{c}{\delta \hat{v}_s(q)}$	0	$\frac{\hat{v}_s(q)}{\delta c}$	$\frac{\hat{v}_s(q)}{\delta c}$
$2s,\ s \in \{H,L\}$	0	$\frac{\delta c}{\hat{v}_s(q)}$	0		
		$\gamma = DC,\ \delta < 1$			
$1L$	0	$\frac{c}{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}$	0	$\frac{[1-\hat{\mu}(q)]\hat{v}_L(q)}{c}$	$\frac{\bar{v}}{c}$
$1H$	0	0	$\frac{c}{\delta\hat{\mu}(q)\hat{v}_H(q)}$		
2	$1-\delta$	$\frac{\delta c}{\hat{v}_L(q)}$	$\frac{\delta c}{\hat{v}_H(q)}$		
		$\gamma = DC,\ 1 \leq \delta \leq \frac{1}{\hat{\mu}(q)}$			
$1L$	$\frac{1}{1-\hat{\mu}(q)}\left(1-\frac{1}{\delta}\right)$	$\frac{c}{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}$	0	$\left[1-\hat{\mu}(q)\delta\right]\frac{\hat{v}_L(q)}{\delta c}$	$\frac{\hat{v}_L(q)}{\delta c} + \frac{\hat{\mu}(q)}{c}[\hat{v}_H(q) - \hat{v}_L(q)]$
$1H$	0	0	$\frac{c}{\delta\hat{\mu}(q)\hat{v}_H(q)}$		
2	0	$\frac{\delta c}{\hat{v}_L(q)}$	$\frac{\delta c}{\hat{v}_H(q)}$		
		$\gamma = DC,\ \delta > \frac{1}{\hat{\mu}(q)}$			
$1L$	1	0	0	$\frac{\hat{v}_H(q)}{\delta c}$	$\frac{\hat{v}_H(q)}{\delta c}$
$1H$	$1-\frac{1}{\delta\hat{\mu}(q)}$	$\frac{c}{\delta\hat{\mu}(q)\hat{v}_H(q)}$	0		
2	0	$\frac{\delta c}{\hat{v}_H(q)}$	0		

Table 1: Equilibrium Bidding Strategies.

$TE(\gamma,\delta;c)$ and the expected winner's effort $WE(\gamma,\delta;c)$. We present the results in Table 2, with functions $\mathcal{W}_1(\cdot)$, $\mathcal{W}_2(\cdot)$, and $\mathcal{W}_3(\cdot)$ to be defined as follows:

$$\begin{aligned}
\mathcal{W}_1(u,z,d;c) &:= -\left(u^3 z + 1\right) d^2 c + \left\{u^2 z(6-u) + 5\right\} dc, \\
\mathcal{W}_2(u,z,d;c) &:= \frac{-d^3 u^2 z\left[u(1+d) - 6\right] + 5d - 1}{d^2} c, \\
\mathcal{W}_3(d;c) &:= \frac{5d-1}{d^2} c.
\end{aligned}$$

The case with $\gamma = CD$ is omitted, since a contest scheme $(CD, 1/\delta)$ is outcome equivalent to (DC,δ) with symmetric players. The results in Table 2 pave the way for our analysis of the optimal contest design.

It is noteworthy that for a given scoring bias $\delta > 0$, symmetric disclosure schemes—$\gamma = CC$ and DD—generate the same ex ante equilibrium outcome—i.e., $TE(CC,\delta;c) =$

	$TE(\gamma,\delta;c)$	$WE(\gamma,\delta;c)$
$\gamma = DD$ or CC, $\delta < 1$	$\frac{\delta\bar{v}}{c}$	$\frac{\delta\bar{v}(5-\delta)}{6c}$
$\gamma = DD$ or CC, $\delta \geq 1$	$\frac{\bar{v}}{\delta c}$	$\frac{\bar{v}(5\delta-1)}{6c\delta^2}$
$\gamma = DC$, $\delta < 1$	$\frac{\delta\left(\hat{v}_L(q)+\hat{\mu}(q)^2\hat{v}_H(q)-\hat{\mu}(q)^2\hat{v}_L(q)\right)}{c}$	$\frac{\hat{v}_L(q)}{6c^2}\mathcal{W}_1\left(\hat{\mu}(q), \frac{\hat{v}_H(q)-\hat{v}_L(q)}{\hat{v}_L(q)}, \delta; c\right)$
$\gamma = DC$, $1 \leq \delta \leq \frac{1}{\hat{\mu}(q)}$	$\frac{1}{c}\left[\frac{\hat{v}_L(q)}{\delta} + \delta\hat{\mu}(q)^2(\hat{v}_H(q) - \hat{v}_L(q))\right]$	$\frac{\hat{v}_L(q)}{6c^2}\mathcal{W}_2\left(\hat{\mu}(q), \frac{\hat{v}_H(q)-\hat{v}_L(q)}{\hat{v}_L(q)}, \delta; c\right)$
$\gamma = DC$, $\delta > \frac{1}{\hat{\mu}(q)}$	$\frac{\hat{v}_H(q)}{\delta c}$	$\frac{\hat{v}_H(q)}{6c^2}\mathcal{W}_3\left(\delta; c\right)$

Table 2: Expected Total Effort and the Expected Winner's Effort in Equilibrium.

$TE(DD,\delta;c)$ and $WE(CC,\delta;c) = WE(DD,\delta;c)$.

3.2 Optimal Contest

The solutions of equilibrium expected total effort and the expected winner's effort enable analysis of the optimum.

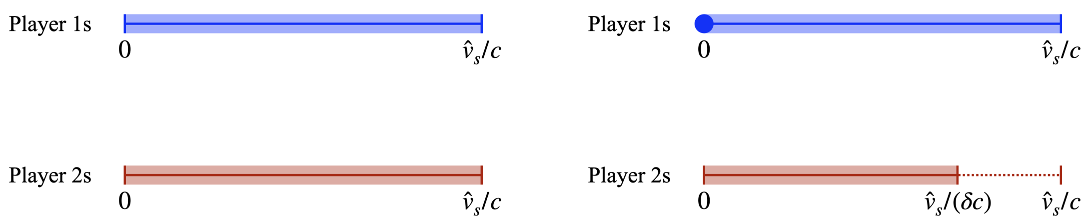
Proposition 1 ***(Optimal Contest)*** *Fix $q \in (1/2, 1]$ and suppose $c_1 = c_2 = c > 0$. The following statements hold.*

(i) If the designer aims to maximize expected total effort, then both $(\gamma^*_{TE}, \delta^*_{TE}) = (CC, 1)$ and $(\gamma^*_{TE}, \delta^*_{TE}) = (DD, 1)$ are optimal.

(ii) If the designer aims to maximize the expected winner's effort, then in the case with $\hat{\mu}(q)\hat{v}_H(q) > 4\hat{v}_L(q)$, both $(\gamma^*_{WE}, \delta^*_{WE}) = \left(CD, \hat{\mu}(q)\right)$ and $(\gamma^*_{WE}, \delta^*_{WE}) = \left(DC, 1/\hat{\mu}(q)\right)$ are optimal; in the case with $\hat{\mu}(q)\hat{v}_H(q) \leq 4\hat{v}_L(q)$, both $(\gamma^*_{WE}, \delta^*_{WE}) = (CC, 1)$ and $(\gamma^*_{WE}, \delta^*_{WE}) = (DD, 1)$ are optimal.

Proposition 1(i) is intuitive and echoes the conventional wisdom of the contest literature. The contest maintains symmetry with a neutral scoring rule $\delta = 1$, as well as symmetric disclosure—i.e., $\gamma \in \{CC, DD\}$. However, Proposition 1(ii) shows that to maximize the expected winner's effort, the designer may deliberately create ex post dual asymmetry between players: She *tilts* the playing field by awarding an information advantage to one player, while *releveling* the playing field by biasing the scoring rule in favor of the other. A *tilting-and-releveling* contest, $\left(CD, \hat{\mu}(q)\right)$ or $\left(DC, 1/\hat{\mu}(q)\right)$, is optimal when the condition $\hat{\mu}(q)\hat{v}_H(q) > 4\hat{v}_L(q)$ is met.

To interpret the result, it is useful to first understand the bidding equilibrium under symmetric disclosure vis-à-vis that under asymmetric disclosure, which is summarized in Figure 1.



(a) $\delta = 1$. (b) $\delta > 1$.

Figure 1: Equilibrium Strategies with Symmetric Players: $\gamma = DD$.

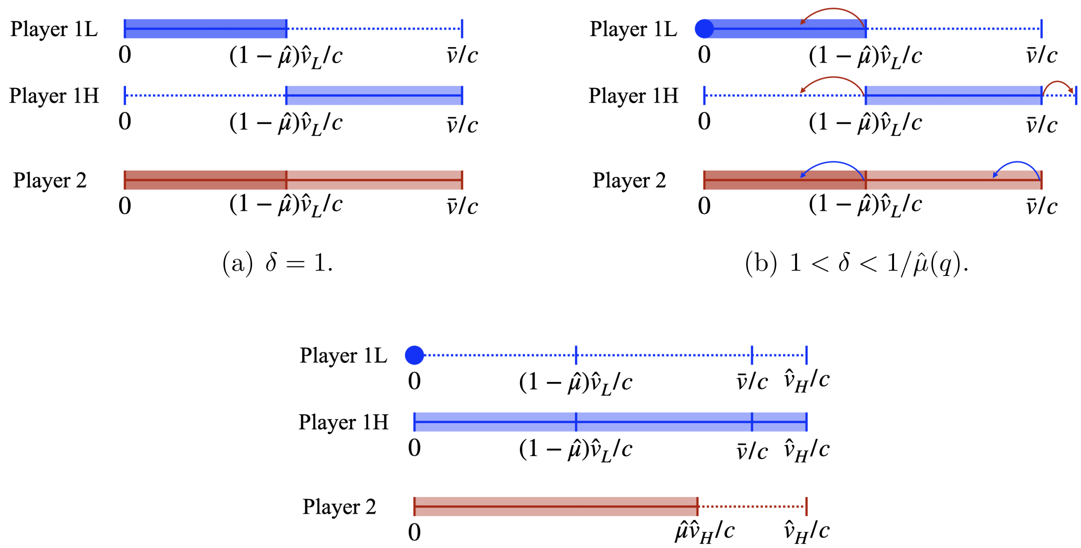
Equilibrium under Symmetric Disclosure The equilibrium under a symmetric disclosure scheme with discrete signal spaces resembles that in a standard complete-information all-pay auction. With $\delta = 1$, a player's effort is uniformly distributed over the interval $[0, \hat{v}_s(q)/c]$ under DD, where $\hat{v}_s(q)$ is the updated expected prize value upon receiving a signal $s \in \{H, L\}$. Analogously, one's effort under CC is uniformly distributed over $[0, \bar{v}/c]$. A symmetric contest, $(DD, 1)$ or $(CC, 1)$, fully extracts the players' surplus and maximizes expected total effort.

Suppose instead that a biased scoring rule is in place, e.g., $\delta > 1$. Player 2 secures a sure win by bidding $\hat{v}_s(q)/(\delta c)$ under DD (or $\bar{v}/(\delta c)$ under CC), which leaves him with positive surplus. The handicapped player 1 continues to bid up to $\hat{v}_s(q)/c$ under DD (or $\bar{v}/c$ under CC), but he now stays inactive—i.e., exerting zero effort—with a positive probability. The biased scoring rule is obviously suboptimal. We illustrate this rationale in Figure 1 for the case of $\gamma = DD$.

Equilibrium under Asymmetric Disclosure Asymmetric disclosure fundamentally changes the nature of the equilibrium. Assuming $\gamma = DC$ and $\delta = 1$, we illustrate players' equilibrium bidding strategies in Figure 2(a). Player 1 is informed, and his equilibrium bidding strategy is signal-dependent. Player 1, upon receiving signal L, is referred to as player $1L$; his efforts are uniformly distributed on $\big[0, [1 - \hat{\mu}(q)]\hat{v}_L(q)/c\big]$, while those of player $1H$ are distributed on $\big[[1 - \hat{\mu}(q)]\hat{v}_L(q)/c, \bar{v}/c\big]$ (see Table 1). Player 2's efforts are distributed over the interval $\big[0, \bar{v}/c\big]$.[8]

Player $1L$—due to his lower updated expected prize valuation—is effectively an underdog when competing with the uninformed player 2. The distribution of his efforts includes zero, which implies a zero equilibrium payoff for him. In contrast, player $1H$ has a higher expected

[8]The bidding supports of players $1L$ and $1H$ are disjoint and separated by the cutoff $[1 - \hat{\mu}(q)]\hat{v}_L(q)/c$. The distribution of x_2, however, has different densities for efforts above and below the cutoff. This occurs because in this common-value all-pay auction, the uninformed player 2 takes into account and strategically responds to player 1's type-dependent bidding strategy when placing his bid.



(a) $\delta = 1$.

(b) $1 < \delta < 1/\hat{\mu}(q)$.

(c) $\delta = 1/\hat{\mu}(q)$.

Figure 2: Equilibrium Strategies with Symmetric Players: $\gamma = DC$.

prize valuation and becomes a favorite vis-à-vis player 2. The upper support of his efforts remains at $\bar{v}/c$, although he can bid up to $\hat{v}_H(q)/c$. He has no incentive to bid more than $\bar{v}/c$ because player 2's effort is capped at that level.

The contest $(DC, 1)$ is obviously suboptimal. This naturally prompts the question of how player $1H$ can be further incentivized to bid more than $\bar{v}/c$, which inspires tilting and releveling.

Tilting and Releveling Raising δ above 1 incentivizes player $1H$. We illustrate this rationale in Figure 2(b). A scoring bias $\delta > 1$ favors player 2 and discourages player $1L$. In contrast, although player $1H$ continues to enjoy the upper hand for δ in the range of $\left[1, 1/\hat{\mu}(q)\right]$, effort $\bar{v}/c$ no longer guarantees a sure win. Thus the unfavorable scoring rule compels him to step up his effort: The upper support of his effort increases with δ.

Tilting and releveling "gives up" the low-type informed player 1, but could benefit from his better-incentivized high-type counterpart, since the latter may bid more than $\bar{v}/c$. This is obviously suboptimal for total-effort maximization, but suggests a potential for elevating the expected winner's effort, in which case only the winner's effort (i.e., the modified first-order statistic) matters. The details are discussed below.

Tilting and Releveling as an Optimal Contest By Proposition 1(ii), with $\gamma = DC$, a bias $\delta = 1/\hat{\mu}(q)$ could maximize the expected winner's effort. Recall that contests under symmetric disclosure, either DD or CC, generate the same ex ante equilibrium outcomes for a given δ. We compare the tilting-and-releveling contest $\big(DC, 1/\hat{\mu}(q)\big)$ with a fully symmetric contest $(CC, 1)$ to elucidate the underlying trade-off.

Under $(CC, 1)$, players maintain their prior, so their efforts are uniformly distributed over $[0, \bar{v}/c]$. Players' equilibrium strategies in the contest $\big(DC, 1/\hat{\mu}(q)\big)$ are illustrated in Figure 2(c). Imagine first that a low signal $s = L$ is realized. The negative shock, together with the unfavorable scoring rule, forces player $1L$ to give up—i.e., with his bidding strategy degenerating to a singleton at zero—which clearly causes a loss compared with the case of $(CC, 1)$. However, player 2 remains uninformed and is immune to the negative shock; he remains active, which provides insurance for the performance of the contest. Then suppose $s = H$. Player 1—because of the upwardly revised prize expectation and the unfavorable scoring rule—may bid more than $\bar{v}/c$, with the upper support reaching $\hat{v}_H(q)/c$. The contest, when maximizing the expected winner's effort, could outperform $(CC, 1)$.

The trade-off between $\big(DC, 1/\hat{\mu}(q)\big)$ and $\big(CC, 1\big)$ ultimately depends on $\hat{\mu}(q)$, $\hat{v}_H(q)$, and $\hat{v}_L(q)$. First, tilting and releveling could yield a gain when a high signal is realized, which occurs with a probability of $\hat{\mu}(q)$. Therefore, the former is more likely to prevail with a large $\hat{\mu}(q)$. Second, the gain of the tilting-and-releveling contest is more significant when the signal prompts substantial upward revision in prize valuation—i.e., from $\bar{v}$ to $\hat{v}_H(q)$—which requires a larger $\hat{v}_H(q)$ relative to $\hat{v}_L(q)$. Summing these leads to the condition $\hat{\mu}(q)\hat{v}_H(q) > 4\hat{v}_L(q)$ for the optimality of $\big(DC, 1/\hat{\mu}(q)\big)$. The scoring bias $\delta = 1/\hat{\mu}(q)$ relevels the contest under $\gamma = DC$ and enables players $1H$ and 2 to win with an equal probability with $s = H$; it also perfectly eliminates the rent afforded to player 1 by his information advantage.

Tilting and releveling may well emerge as the optimum with asymmetric players, which we discuss in Section 3.3.1. The same rationale applies and also determines whom—the stronger or the weaker player—should respectively be awarded the information advantage and favoritism in terms of scoring bias.

Complementarity Between Information Disclosure and Scoring Bias The two instruments, information disclosure and scoring bias, play *complementary* roles. That is, the optimum either requires full symmetry or embraces dual asymmetry. Suppose that the designer is allowed to distort the contest in only dimension, either by setting the disclosure scheme while maintaining a neutral scoring rule or biasing the scoring rule while being constrained by symmetric disclosure. The following ensues.

Remark 1 ***(Unidimensional Contest Design)*** *Fix* $q \in (1/2, 1]$ *and suppose* $c_1 = c_2 = c > 0$*. The following statements hold.*

(i) Fix $\delta = 1$. A symmetric disclosure scheme—i.e., $\gamma \in \{CC, DD\}$—maximizes both expected total effort and the expected winner's effort simultaneously.

(ii) Fix $\gamma \in \{CC, DD\}$. The neutral scoring bias—i.e., $\delta = 1$—maximizes both expected total effort and the expected winner's effort simultaneously.

With $\delta = 1$, an asymmetric disclosure scheme cannot force the high-type player 1 to raise his maximum effort above $\bar{v}/c$, as Figure 2(a) illustrates. Similarly, with a symmetric disclosure scheme, biasing the scoring rule only allows the favored player to slack off, as Figure 1(b) shows. Asymmetry in only one dimension is always suboptimal.

3.3 Discussions and Extensions

In this part, we consider three extensions that shed further light on the principles of optimal contest design that combines the tools of scoring rule and disclosure policy.

3.3.1 Asymmetric Players

We first consider the case of asymmetric players. Equilibrium characterization for this case is provided in Appendix A. Without loss of generality, we assume $c_1 > c_2$, so player 2 is the stronger player. The following ensues.

Proposition 2 ***(Optimal Contest with Asymmetric Players)*** *Fix $q \in (1/2, 1]$. The following statements hold.*

(i) If the designer aims to maximize expected total effort, then both $(\gamma^*_{TE}, \delta^*_{TE}) = (CC, c_2/c_1)$ and $(\gamma^*_{TE}, \delta^*_{TE}) = (DD, c_2/c_1)$ are optimal.

(ii) If the designer aims to maximize the expected winner's effort, then in the case with $\hat{\mu}(q)\hat{v}_H(q) > \left(2\frac{c_2}{c_1} + 2\right)\hat{v}_L(q)$, the optimal scheme is $(\gamma^*_{WE}, \delta^*_{WE}) = \left(DC, \frac{c_2}{\hat{\mu}(q)c_1}\right)$; in the case with $\hat{\mu}(q)\hat{v}_H(q) \leq \left(2\frac{c_2}{c_1} + 2\right)\hat{v}_L(q)$, both $(\gamma^*_{WE}, \delta^*_{WE}) = (CC, c_2/c_1)$ and $(\gamma^*_{WE}, \delta^*_{WE}) = (DD, c_2/c_1)$ are optimal.

Proposition 2(i), again, affirms the conventional wisdom of leveling the playing field. In this case, a "fair" bias $\delta = c_2/c_1$ is required to fully offset the ex ante asymmetry in terms of bidding efficiency. A symmetric disclosure scheme, together with the fair bias, leads to an ex post fully symmetric contest, which fully extracts players' surplus.

Proposition 2(ii) states that a tilting-and-releveling contest $\left(DC, c_2/[\hat{\mu}(q)c_1]\right)$ could be optimal when the designer's objective is to maximize the expected winner's effort. The underdog, player 1, is provided with an information advantage. The bias $\delta = c_2/[\hat{\mu}(q)c_1]$ relevels the competition between players $1H$ and 2—as $1/\hat{\mu}(q)$ does in the symmetric case—and entirely discourages player $1L$. The same trade-off looms large for the designer, as in the case with symmetric players. Notably, the releveling bias $c_2/[\hat{\mu}(q)c_1]$ could remain literally biased against player 2—i.e., $c_2/[\hat{\mu}(q)c_1] < 1$—if players are excessively heterogeneous. However, it is more favorable to the uninformed player 2 relative to the "fair" bias c_2/c_1 that perfectly offsets player 1's information advantage—i.e., $c_2/[\hat{\mu}(q)c_1] > c_2/c_1$.

To understand why the *weaker* player 1 receives an information advantage, recall that in a tilting-and-releveling contest (i) the low-type informed player is fully discouraged—which incurs a loss—and (ii) the uninformed player stays active regardless—which provides insurance. First, giving up the weaker player minimizes the loss, since in any case player 1's higher marginal cost limits the potential of his contribution. Second, keeping the stronger player 2 active maximizes the insurance.

This rationale is further illustrated by the optimal condition $\hat{\mu}(q)\hat{v}_H(q) > [2(c_2/c_1) + 2]\hat{v}_L(q)$. The tilting-and-releveling contest is more likely to prevail when players are more asymmetric (i.e., with a smaller c_2/c_1): The loss incurred when $s = L$ is less significant because the forgone effort of player $1L$ is limited by his relatively higher cost, while the insurance provided by player 2 is large due to his more significant bidding competence.

3.3.2 Credibility of Disclosure Policies

The baseline model assumes that the designer can commit to her announced disclosure policy and abstracts away the issue of *credibility* (Akbarpour and Li, 2020; Lin and Liu, 2024): The commitment power can be called into question, since the designer may find it profitable to deviate from her precommitted prescriptions. We now consider the possibility of deviation. To the best of our knowledge, we are the first to examine the issue of credibility in the contest literature.

Fix a disclosure scheme $\gamma \in \{CC, CD, DC, DD\}$ and let $\gamma(i)$ indicate the specific disclosure to a player $i \in \{1, 2\}$ under γ. For example, $\gamma(1) = C$ and $\gamma(2) = D$ for $\gamma = CD$. Consider a disclosure scheme γ announced by the designer and a potential deviation $\gamma' \neq \gamma$. Assume that player i can detect the deviation if and only if $\gamma(i) \neq \gamma'(i)$. For example, the player detects deviation if he unexpectedly learned about the signal from the designer with $\gamma(i) = C$, or was denied access to it with $\gamma(i) = D$. He maintains his belief about the disclosure to his opponent regardless, since he cannot detect deviation in that respect. Suppose that the designer deviates from her disclosure to one player i, i.e., $\gamma(i) \neq \gamma'(i)$. Player i—upon detecting the deviation—would have an incentive to *complain* if he becomes worse off by adopting a bidding strategy that best responds to his opponent's equilibrium strategy under the announced contest scheme (γ, δ); the contest dissolves when a complaint arises. The player *remains silent* otherwise and the contest proceeds.

A credible contest is formally defined as follows.

Definition 1 ***(Credible Contest)*** *A contest* (γ, δ) *is credible if for every deviation of disclosure policy* $\gamma' \neq \gamma$*, either (i) at least one player who detects it complains, or (ii) every player who detects it remains silent but such a deviation reduces the designer's expected payoff.*[9]

In short, a credible contest prevents profitable deviation, which imposes an additional constraint on our joint design problem. The optimal contest that satisfies this requirement is established as follows.

Proposition 3 ***(Optimal Credible Contest)*** *Fix* $q \in (1/2, 1]$ *and suppose* $c_1 = c_2 = c > 0$*. The following statements hold.*

(i) If the designer aims to maximize expected total effort, then $(\gamma^*_{TE}, \delta^*_{TE}) = (DD, 1)$ *is an optimal credible contest.* $(\gamma^*_{TE}, \delta^*_{TE}) = (CC, 1)$ *is also an optimal credible contest if and only if* $\hat{\mu}(q) \leq 1/2$*.*

(ii) If the designer aims to maximize the expected winner's effort, then

(a) in the case with $\hat{\mu}(q) \leq 5/7$ *and* $\hat{\mu}(q)\hat{v}_H(q) > 4\hat{v}_L(q)$*, both* $(\gamma^*_{WE}, \delta^*_{WE}) = \big(CD, \hat{\mu}(q)\big)$ *and* $(\gamma^*_{WE}, \delta^*_{WE}) = \big(DC, 1/\hat{\mu}(q)\big)$ *are optimal credible contests;*

(b) in the case with $\hat{\mu}(q) > 5/7$ *and* $\{13\hat{\mu}(q) - 18[\hat{\mu}(q)]^2\}\hat{v}_H(q)/\{2[1-\hat{\mu}(q)]\} > \hat{v}_L(q)$*, both* $(\gamma^*_{WE}, \delta^*_{WE}) = \big(CD, 5 - 6\hat{\mu}(q)\big)$ *and* $(\gamma^*_{WE}, \delta^*_{WE}) = \big(DC, 1/[5 - 6\hat{\mu}(q)]\big)$ *are optimal credible contests;*

(c) in all other cases, $(\gamma^*_{WE}, \delta^*_{WE}) = (DD, 1)$ *is an optimal credible contest; so is* $(\gamma^*_{WE}, \delta^*_{WE}) = (CC, 1)$ *if and only if* $\hat{\mu}(q) \leq 1/3$*.*

By Proposition 3(i), $(\gamma^*_{TE}, \delta^*_{TE}) = (DD, 1)$ remains optimal for total effort maximization. The disclosure policy DD is credible: Any deviation can be detected by at least one player; simple analysis can verify that one will complain if he suffers from a loss of information.

[9] Our approach to credible information disclosure is reminiscent of the setup of Lin and Liu (2024) to model credible persuasion, which assumes that a sender's deviation will be detected if it alters the message distribution. Their notion of credibility rules out detectable deviations and requires that a sender not profit from an undetectable deviation. They focus on a sender-receiver problem in which the message is publicly observed, while we consider a game-theoretic environment that allows for private disclosure. Notably, we allow for detectable deviations, provided that they do not reduce the player's payoff—i.e., the player would not file a complaint.

However, the ex ante equivalence of symmetric disclosure schemes CC and DD no longer holds. Suppose that the designer announces CC while disclosing his signal privately to one player. The player would bid $\bar{v} := \mu v_H + (1-\mu)v_L$ to secure a win if $s = H$—which benefits the designer—and bid zero if $s = L$. This deviation benefits the designer if the probability of realizing a high signal is large; the privately informed player also benefits, so he would not complain. These render the announced policy non-credible. As a result, by Proposition 3(i), with $\delta = 1$, $\gamma = CC$ is credible if and only if a high signal is less likely, i.e., $\hat{\mu}(q) \leq 1/2$.

By Proposition 3(ii), to maximize the expected winner's effort, a tilting-and-releveling contest can still prevail. However, the credibility requirement may be a binding constraint. Recall by Proposition 1 that absent credibility concerns, a tilting-and-releveling contest with $(\gamma^*_{WE}, \delta^*_{WE}) = \big(CD, \hat{\mu}(q)\big)$ or $(\gamma^*_{WE}, \delta^*_{WE}) = \big(DC, 1/\hat{\mu}(q)\big)$ is optimal for $\hat{\mu}(q)\hat{v}_H(q) > 4\hat{v}_L(q)$. By Proposition 3(ii), its optimality can be preserved for $\hat{\mu}(q) \leq 5/7$, but dissolves when the probability of a high signal $\hat{\mu}(q)$ is high.

To satisfy the credibility requirement, the scoring rule for a tilting-and-releveling contest has to favor the uninformed player further: The uninformed player would be excessively privileged if the designer deviates and awards him the signal, in which case the designer would be worse off.[10] This altered tilting-and-releveling contest emerges in the optimum in case (b) of Proposition 3(ii). Alternatively, the designer can simply feed the signal to both players and resort to a neutral scoring rule, in which case a fully symmetric contest $(DD, 1)$ arises, as in case (c) of Proposition 3(ii).

3.3.3 Expected Maximum Effort

With a scoring bias $\delta \neq 1$, the winner of the contest may not be the one who contributes the highest effort. Maximizing the expected winner's effort presumes that the designer benefits only from the winning entry, which is plausible when the designer cannot separate the prize allocation from the adoption of contestants' output—e.g., admissions contests at universities or competitions for promotions within firms. In some contexts, the designer may award the prize based on her preferred rules, yet choose to use a higher-quality entry from another contestant; for instance, Netflix did not adopt the algorithm submitted by the winner of its Netflix Prize.[11] We now allow the designer to maximize the expected maximum effort of the contest, denoted by $ME(\gamma, \delta; c)$.

A general analysis is challenging. However, together with numerical exercises, we can verify that the optimal contest is either a symmetric contest $(CC, 1)$ or a tilting-and-releveling contest $(CD, \hat{\mu}(q))$, or equivalently, $(DC, 1/\hat{\mu}(q))$. Comparing $(CC, 1)$ with $(CD, \hat{\mu}(q))$ leads

[10] It is straightforward to verify $1/[5 - 6\hat{\mu}(q)] > 1/\hat{\mu}(q)$ for $\hat{\mu}(q) > 5/7$.

[11] See `tinyurl.com/37kdtz74`.

to the following.

Proposition 4 ***(Optimal Expected-maximum-effort-maximizing Contests)*** *Fix $q \in (1/2, 1]$ and suppose $c_1 = c_2 = c > 0$. Consider two contests $(CC, 1)$ and $\big(CD, \hat{\mu}(q)\big)$. The former generates a higher expected maximum effort than the latter if and only if*

$$\frac{\hat{v}_L(q)}{\hat{v}_H(q)} > \frac{\hat{\mu}(q)\left[2 - \hat{\mu}(q)\right]}{4}.$$

Proposition 4 thus establishes the sufficient and necessary condition for an optimal tilting-and-releveling contest.

4 General Value Distribution and Endogenous Information Structure for Disclosure

In this section, we extend the model to allow for multiple value states and let the designer flexibly design the information structure of her investigation. Specifically, suppose that the common value for the prize v is distributed on the set $\{v_1, v_2, \ldots, v_K\}$ with $K \geq 2$ and $\mu_k := \Pr(v = v_k) > 0$ for $k \in \{1, 2, \ldots, K\}$. Without loss of generality, assume that $0 < v_1 < v_2 < \cdots < v_K$. Again, we denote by $\bar{v}$ the ex ante expected prize value. The designer has full control over the amount of information to be revealed and the form of the signal disclosed to players. This corresponds to the concept of Bayesian persuasion (Kamenica and Gentzkow, 2011). An information structure comprises a signal space $\mathcal{S}$ and a collection of likelihood distributions $\pi(\cdot|v)$ over $\mathcal{S}$.[12] The designer sets $\mathcal{S}$ and $\big(\gamma, \delta, \pi(\cdot|v)\big)$.[13]

A fully symmetric contest, $(CC, 1)$ or $(DD, 1)$, fully dissipates the rent, thereby maximizing total effort. We thus focus on the maximization of the expected winner's effort. Fixing $\gamma \in \{CC, DD\}$, as in the baseline case, it is straightforward to verify that $\delta = 1$ maximizes the expected winner's effort, which equals $\frac{2}{3}\bar{v}$. For the case of $\gamma = DC$, the following lemma establishes the optimality of binary signal spaces, which simplifies the joint design problem.

Lemma 1 ***(Optimality of Binary Signals)*** *Fix $\gamma = DC$ and $\delta > 0$. An information structure with binary signals—i.e., $\mathcal{S} = \{H, L\}$—maximizes the expected winner's effort.*

[12] For example, the information structure described in Section 2 involves a binary signal space $\mathcal{S} = \{H, L\}$ and a conditional likelihood distribution corresponding to each underlying state—i.e., v_H or v_L—parameterized by q [see Equation (1)].

[13] It is noteworthy that this remains a limited information design exercise. Addressing a fully general information design problem in our context is technically demanding, because potential correlation between signals would significantly complicate the analysis of common-value all-pay auctions.

In principle, the optimal information structure may require up to K signals if the value state space is K-dimensional (see, e.g., Arieli, Babichenko, Smorodinsky, and Yamashita, 2023). Lemma 1 states that two signals suffice within our context (see Appendix C for more details). Specifically, we show in the proof that holding $\gamma = DC$ and $\delta > 0$ fixed, for any information structure with a signal space that contains three or more elements, we can construct an alternative information structure with one fewer signal to attain a weakly higher expected winner's effort. This allows us to restrict attention to binary signal spaces, which significantly reduces the dimensionality of the information design problem and enables a closed-form characterization of the optimum.

Given a binary signal space $\mathcal{S} = \{H, L\}$, denote by v_s^π the expected prize value conditional on s, i.e., $\mathbb{E}(v|s)$. Without loss of generality, assume that a realization of $s = H$ gives rise to a higher expected prize value, i.e., $v_H^\pi \geq v_L^\pi$. In addition, define $\mu^\pi := \Pr(s = H)$.

In our context, designing the information structure $\pi(\cdot|v)$ with $\mathcal{S} = \{H, L\}$ is equivalent to choosing a distribution of posterior expectations, $(v_H^\pi, v_L^\pi, \mu^\pi)$, subject to the constraint that the distribution can be induced by a binary signal structure. Denote the cumulative distribution functions of v and v^π by $F(x)$ and $G(x)$, respectively. It is well known in the literature that we can find a binary signal structure that generates $(v_H^\pi, v_L^\pi, \mu^\pi)$ if and only if the following conditions are satisfied (see, e.g., Gentzkow and Kamenica, 2016; Kolotilin, 2018).

$$\int_0^t F(x)dx \geq \int_0^t G(x)dx \text{ for } v_1 \leq t \leq v_K, \text{ and} \tag{2}$$

$$\mu^\pi v_H^\pi + (1 - \mu^\pi) v_L^\pi = \bar{v}. \tag{3}$$

The following proposition fully characterizes the optimal contest.

Proposition 5 ***(Optimal Contest with Multiple Value States and an Endogenous Information Structure)*** *Suppose $c_1 = c_2 = c > 0$. Consider the joint design of scoring bias $\delta > 0$, disclosure scheme γ, and information structure $\pi(\cdot|v)$. If the designer aims to maximize the expected winner's effort, the following holds.*

(i) In the case with $\bar{v}/v_1 > 4$, the optimal contest consists of $(\gamma_{WE}^*, \delta_{WE}^*) = \big(DC, 1/\mu^\pi\big)$,

or equivalently, $\big(CD, \mu^\pi\big)$, *with*

$$\mu^\pi = \min\left\{\sum_{\ell=k^*}^{K} \mu_\ell, \mu^*(k^*)\right\},$$

$$v_H^\pi = \frac{[\mu^\pi - \Pr(v > v_{k^*})]v_{k^*} + \Pr(v > v_{k^*})\mathbb{E}[v|v > v_{k^*}]}{\mu^\pi}, \text{ and}$$

$$v_L^\pi = \frac{[1 - \mu^\pi - \Pr(v \le v_{k^*})]v_{k^*} + \Pr(v \le v_{k^*})\mathbb{E}[v|v \le v_{k^*}]}{1 - \mu^\pi},$$

where $\mu^*(k) := 3 - \frac{\sum_{\ell=1}^{k}\mu_\ell}{2} - \frac{\sum_{\ell=k+1}^{K}\mu_\ell v_\ell}{2v_k}$, *and* $k^* := \min\left\{k : \mu^*(k) \ge \sum_{\ell=k+1}^{K}\mu_\ell\right\}$. *This distribution of posterior expectations is achieved by a signal structure with* $\pi(H|v) = 0$ *for* $v < v_{k^*}$, $\pi(H|v) = \frac{\mu^\pi - \Pr(v > v_{k^*})}{\mu_{k^*}}$ *for* $v = v_{k^*}$, *and* $\pi(H|v) = 1$ *for* $v > v_{k^*}$.

(ii) *In the case with* $\bar{v}/v_1 \le 4$, *both* $(\gamma_{WE}^*, \delta_{WE}^*) = (CC, 1)$ *and* $(\gamma_{WE}^*, \delta_{WE}^*) = (DD, 1)$, *with an arbitrary information structure* $\pi(\cdot|v)$, *are optimal.*

The implications of our baseline model remain intact. The designer may, again, tilt and relevel to maximize the expected winner's effort, provided that the condition $\bar{v}/v_1 > 4$ is met. The optimal information structure takes a simple form: There exists a threshold value state v_{k^*} such that the designer sends a low signal if $v < v_{k^*}$ and a high signal if $v > v_{k^*}$. For $v = v_{k^*}$, the designer may randomize between the two signals.

5 Concluding Remarks

This paper studies the optimal design of a contest in which two players compete for a common-valued prize. The designer chooses a combination of two instruments: an information disclosure scheme and a scoring bias. Fully symmetric contests—symmetrically disclosed information and a neutral scoring rule—maximize expected total effort, which embraces the conventional wisdom of leveling the playing field. However, when maximizing the expected winner's effort, the contest may feature dual asymmetry that distorts the contest in both dimensions: The designer discloses the signal privately to one player, while a favorable scoring rule compensates the other. Such *tilting-and-releveling* contests could prevail even if the players are ex ante identical.

Our paper is one of the first in the contest literature to examine the optimal combination of multiple design instruments. We demonstrate the complementarity between the instruments, in that the optimum requires either ex post full symmetry or dual asymmetry.

Our results generate novel implications for contest design and shed fresh light on the debate regarding the relationship between (a)symmetry and the performance of a contest.

For future research, it would be promising to revisit our research question within the context of all-pay auction models in more general settings, such as those that involve nonlinear cost functions or multiple players. Also, the information design exercise could be expanded by allowing for correlated signals. Although these extensions are technically challenging, they clearly merit further exploration.

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Appendix A Equilibrium Analysis

In this appendix, we first characterize the equilibrium with (possibly) asymmetric players under an arbitrary contest scheme (γ, δ), with $\gamma \in \{CC, CD, DC, DD\}$ and $\delta > 0$. We then calculate the resulting expected total effort and the expected winner's effort. Our analysis is adapted from Siegel (2014), who provides the technique for the case with a neutral scoring rule $\delta = 1$; here we allow for a scoring bias.

We describe by a function $b_{is}(x; \gamma, \delta)$ the equilibrium bidding strategy of a player i of type s, i.e., when receiving a signal $s \in \{H, L\}$: $b_{is}(0; \gamma, \delta, q)$ gives the probability that player i chooses zero effort—i.e., $x = 0$—and stays inactive, while $b_{is}(x; \gamma, \delta, q)$ provides the probability density of exerting an effort $x > 0$. We omit the subscript s if player $i \in \{1, 2\}$ is not granted access to the signal s, so his equilibrium bidding strategy is given by $b_i(x; \gamma, \delta)$.

A.1 Equilibrium Results

We first characterize the equilibrium under a symmetric information disclosure scheme, i.e., $\gamma \in \{CC, DD\}$, in which case neither player possesses information favoritism.

Proposition A1 ***(Equilibrium Characterization under Symmetric Disclosure)*** *Under $\gamma = DD$, the contest game generates a unique equilibrium, which can be characterized as follows:*

(i) If $\delta < \frac{c_2}{c_1}$, then

$$b_{1s}(x; DD, \delta) = \begin{cases} \frac{c_2}{\delta \hat{v}_s(q)}, & \textit{if } 0 < x \leq \frac{\delta \hat{v}_s(q)}{c_2}, \\ 0, & \textit{otherwise}, \end{cases}$$

$$b_{2s}(x; DD, \delta) = \begin{cases} 1 - \frac{\delta c_1}{c_2}, & \textit{if } x = 0, \\ \frac{\delta c_1}{\hat{v}_s(q)}, & \textit{if } 0 < x \leq \frac{\hat{v}_s(q)}{c_2}, \\ 0, & \textit{otherwise}. \end{cases}$$

(ii) If $\delta \geq \frac{c_2}{c_1}$, then

$$b_{1s}(x; DD, \delta) = \begin{cases} 1 - \frac{c_2}{\delta c_1}, & \textit{if } x = 0, \\ \frac{c_2}{\delta \hat{v}_s(q)}, & \textit{if } 0 < x \leq \frac{\hat{v}_s(q)}{c_1}, \\ 0, & \textit{otherwise}, \end{cases}$$

$$b_{2s}(x; DD, \delta) = \begin{cases} \frac{\delta c_1}{\hat{v}_s(q)}, & \textit{if } 0 < x \leq \frac{\hat{v}_s(q)}{\delta c_1}, \\ 0, & \textit{otherwise}. \end{cases}$$

(iii) The equilibrium bidding strategy under $\gamma = CC$, denoted by $b_i(x; CC, \delta)$, can be obtained by replacing $\hat{v}_s(q)$ with $\bar{v} \equiv \mu v_H + (1-\mu)v_L$ in $b_{is}(x; DD, \delta)$.

Next, we consider the equilibrium under each asymmetric disclosure scheme, i.e., $\gamma = CD$ or DC, in which case one player receives the signal privately.

Proposition A2 ***(Equilibrium Characterization under Asymmetric Disclosure)*** *Under $\gamma = DC$, the contest game generates a unique equilibrium, which can be characterized as follows:*

(i) If $\delta < \frac{c_2}{c_1}$, then

$$b_{1L}(x; DC, \delta) = \begin{cases} \frac{c_2}{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}, & \text{if } 0 < x \leq \frac{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}{c_2}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_{1H}(x; DC, \delta) = \begin{cases} \frac{c_2}{\delta\hat{\mu}(q)\hat{v}_H(q)}, & \text{if } \frac{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}{c_2} < x \leq \frac{\delta\bar{v}}{c_2}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_2(x; DC, \delta) = \begin{cases} 1 - \frac{\delta c_1}{c_2}, & \text{if } x = 0, \\ \frac{\delta c_1}{\hat{v}_L(q)}, & \text{if } 0 < x \leq \frac{[1-\hat{\mu}(q)]\hat{v}_L(q)}{c_2}, \\ \frac{\delta c_1}{\hat{v}_H(q)}, & \text{if } \frac{[1-\hat{\mu}(q)]\hat{v}_L(q)}{c_2} < x \leq \frac{\bar{v}}{c_2}, \\ 0, & \text{otherwise.} \end{cases}$$

(ii) If $\frac{c_2}{c_1} \leq \delta \leq \frac{c_2}{\hat{\mu}(q)c_1}$, then

$$b_{1L}(x; DC, \delta) = \begin{cases} \frac{1}{1-\hat{\mu}(q)}\left(1 - \frac{c_2}{\delta c_1}\right), & \text{if } x = 0, \\ \frac{c_2}{\delta[1-\hat{\mu}(q)]\hat{v}_L(q)}, & \text{if } 0 < x \leq \left[1 - \hat{\mu}(q)\frac{\delta c_1}{c_2}\right]\frac{\hat{v}_L(q)}{c_1}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_{1H}(x; DC, \delta) = \begin{cases} \frac{c_2}{\delta\hat{\mu}(q)\hat{v}_H(q)}, & \text{if } \left[1 - \hat{\mu}(q)\frac{\delta c_1}{c_2}\right]\frac{\hat{v}_L(q)}{c_1} < x \leq \frac{\hat{v}_L(q)}{c_1} + \frac{\delta\hat{\mu}(q)[\hat{v}_H(q)-\hat{v}_L(q)]}{c_2}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_2(x; DC, \delta) = \begin{cases} \frac{\delta c_1}{\hat{v}_L(q)}, & \text{if } 0 < x \leq \left[1 - \hat{\mu}(q)\frac{\delta c_1}{c_2}\right]\frac{\hat{v}_L(q)}{\delta c_1}, \\ \frac{\delta c_1}{\hat{v}_H(q)}, & \text{if } \left[1 - \hat{\mu}(q)\frac{\delta c_1}{c_2}\right]\frac{\hat{v}_L(q)}{\delta c_1} < x \leq \frac{\hat{v}_L(q)}{\delta c_1} + \frac{\hat{\mu}(q)[\hat{v}_H(q)-\hat{v}_L(q)]}{c_2}, \\ 0, & \text{otherwise.} \end{cases}$$

(iii) If $\delta > \frac{c_2}{\hat{\mu}(q)c_1}$, *then*

$$b_{1L}(x; DC, \delta) = \begin{cases} 1, & \text{if } x = 0, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_{1H}(x; DC, \delta) = \begin{cases} 1 - \frac{c_2}{\delta c_1 \hat{\mu}(q)}, & \text{if } x = 0, \\ \frac{c_2}{\delta \hat{\mu}(q) \hat{v}_H(q)}, & \text{if } 0 < x \leq \frac{\hat{v}_H(q)}{c_1}, \\ 0, & \text{otherwise,} \end{cases}$$

$$b_2(x; DC, \delta) = \begin{cases} \frac{\delta c_1}{\hat{v}_H(q)}, & \text{if } 0 < x \leq \frac{\hat{v}_H(q)}{\delta c_1}, \\ 0, & \text{otherwise.} \end{cases}$$

The equilibrium under (CD, δ) *can be obtained similarly.*

A.2 Expected Total Effort and the Expected Winner's Effort

Propositions A1 and A2 lead to the following.

Lemma A1 ***(Expected Total Effort under Different Contest Schemes)*** *Fixing a contest scheme* (δ, γ) *and a profile of marginal effort costs* (c_1, c_2), *the contest generates an equilibrium expected total effort*

$$TE(CC, \delta; c_1, c_2) = TE(DD, \delta; c_1, c_2) = \begin{cases} \frac{\delta \bar{v}(c_1+c_2)}{2c_2^2}, & \text{if } \delta < \frac{c_2}{c_1}, \\ \frac{\bar{v}(c_1+c_2)}{2\delta c_1^2}, & \text{if } \delta \geq \frac{c_2}{c_1} \end{cases}$$

for symmetric disclosure schemes. Under asymmetric disclosure schemes, the equilibrium expected total effort of the contest can be obtained as

$$\begin{aligned} TE(DC, \delta; c_1, c_2) &= TE(CD, 1/\delta; c_2, c_1) \\ &= \begin{cases} \frac{\delta(c_1+c_2)\left(\hat{v}_L(q)+\hat{\mu}(q)^2\hat{v}_H(q)-\hat{\mu}(q)^2\hat{v}_L(q)\right)}{2c_2{}^2}, & \text{if } \delta < \frac{c_2}{c_1}, \\ \frac{c_1+c_2}{2c_1c_2}\left[\frac{c_2}{\delta c_1}\hat{v}_L(q) + \frac{\delta c_1}{c_2}\hat{\mu}(q)^2(\hat{v}_H(q) - \hat{v}_L(q))\right], & \text{if } \frac{c_2}{c_1} \leq \delta \leq \frac{c_2}{\hat{\mu}(q)c_1}, \\ \frac{(c_1+c_2)\hat{v}_H(q)}{2\delta c_1^2}, & \text{if } \delta > \frac{c_2}{\hat{\mu}(q)c_1}. \end{cases} \end{aligned}$$

Further, we derive the equilibrium expected winner's efforts. The following ensues.

Lemma A2 ***(Expected Winner's Effort under Different Contest Schemes)*** *Fixing a contest scheme* (δ, γ) *and a profile of marginal effort costs* (c_1, c_2), *the equilibrium expected* *winner's effort from the contest game is*

$$WE(CC,\delta;c_1,c_2) = WE(DD,\delta;c_1,c_2) = \begin{cases} \frac{\delta\bar{v}(2c_1+3c_2-c_1\delta)}{6{c_2}^2}, & \textit{if } \delta < \frac{c_2}{c_1}, \\ \frac{\bar{v}(3c_1\delta - c_2 + 2c_2\delta)}{6{c_1}^2\delta^2}, & \textit{if } \delta \geq \frac{c_2}{c_1}, \end{cases}$$

and

$$WE(DC,\delta;c_1,c_2) = \begin{cases} \frac{\hat{v}_L(q)}{6c_1c_2}\mathcal{W}_1\left(\hat{\mu}(q), \frac{\hat{v}_H(q)-\hat{v}_L(q)}{\hat{v}_L(q)}, \frac{\delta c_1}{c_2}; c_1, c_2\right), & \textit{if } \delta < \frac{c_2}{c_1}, \\ \frac{\hat{v}_L(q)}{6c_1c_2}\mathcal{W}_2\left(\hat{\mu}(q), \frac{\hat{v}_H(q)-\hat{v}_L(q)}{\hat{v}_L(q)}, \frac{\delta c_1}{c_2}; c_1, c_2\right), & \textit{if } \frac{c_2}{c_1} \leq \delta \leq \frac{c_2}{\hat{\mu}(q)c_1}, \\ \frac{\hat{v}_H(q)}{6c_1c_2}\mathcal{W}_3\left(\frac{\delta c_1}{c_2}; c_1, c_2\right), & \textit{if } \delta > \frac{c_2}{\hat{\mu}(q)c_1}, \end{cases}$$

where $\mathcal{W}_1(\cdot,\cdot,\cdot)$, $\mathcal{W}_2(\cdot,\cdot,\cdot)$, and $\mathcal{W}_3(\cdot)$ are defined as follows:

$$\begin{aligned} \mathcal{W}_1(u,z,d;c_1,c_2) &:= -c_2\left(u^3z+1\right)d^2 + \left\{u^2z\left[3(c_1+c_2)-c_1u\right]+2c_1+3c_2\right\}d, \\ \mathcal{W}_2(u,z,d;c_1,c_2) &:= \frac{d^3\left(-u^2\right)z\left[u(c_1+c_2d)-3(c_1+c_2)\right]+3c_1d-c_1+2c_2d}{d^2}, \\ \mathcal{W}_3(d;c_1,c_2) &:= \frac{c_1(3d-1)+2c_2d}{d^2}. \end{aligned}$$

Moreover, we have that $WE(CD,\delta;c_1,c_2) = WE(DC,1/\delta;c_2,c_1)$.

Lemmas A1 and A2 pave the way for our analysis of the optimal contest design.

A.3 Proofs of Propositions A1 and A2 and Lemmas A1 and A2

Proof. It can be verified that the strategy profiles provided in Propositions A1 and A2 constitute an equilibrium under $\gamma \in \{CC, DD\}$ and $\gamma \in \{DC, CD\}$, respectively. The equilibrium uniqueness in Proposition A1 follows from Hillman and Riley (1989) and Baye, Kovenock, and De Vries (1996), and that in Proposition A2 follows from Siegel (2014). Lemmas A1 and A2 follow immediately from the equilibrium characterizations in Propositions A1 and A2. ■

Appendix B Proofs

Proofs of Tables 1 and 2

Proof. See Appendix A. ■

Proof of Proposition 1

Proof. See the proof of Proposition 2. ■

Proof of Proposition 2

Proof. We first prove part (i) of the proposition. From Table 2, it is straightforward to verify that $\delta = \frac{c_2}{c_1}$ maximizes $TE(CC, \delta; c_1, c_2)$ and $TE(DD, \delta; c_1, c_2)$, and the maximum expected total effort is $\frac{(c_1+c_2)\bar{v}}{2c_1c_2}$. Similarly, from Table 2, it can be verified that either $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{c_2}{\hat{\mu}(q)c_1}$ maximizes $TE(DC, \delta; c_1, c_2)$. Moreover, we have that

$$
\begin{aligned}
TE\left(DC, \frac{c_2}{c_1}; c_1, c_2\right) &= \frac{(c_1+c_2)\left\{\hat{\mu}^2(q)\hat{v}_H(q) + \left[1-\hat{\mu}^2(q)\right]\hat{v}_L(q)\right\}}{2c_1c_2} \\
&< \frac{(c_1+c_2)\left\{\hat{\mu}(q)\hat{v}_H(q) + \left[1-\hat{\mu}(q)\right]\hat{v}_L(q)\right\}}{2c_1c_2} \\
&= \frac{(c_1+c_2)\bar{v}}{2c_1c_2} = TE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right),
\end{aligned}
$$

and

$$
\begin{aligned}
TE\left(DC, \frac{c_2}{\hat{\mu}(q)c_1}; c_1, c_2\right) &= \frac{(c_1+c_2)\hat{\mu}(q)\hat{v}_H(q)}{2c_1c_2} \\
&< \frac{(c_1+c_2)\left\{\hat{\mu}(q)\hat{v}_H(q) + \left[1-\hat{\mu}(q)\right]\hat{v}_L(q)\right\}}{2c_1c_2} \\
&= \frac{(c_1+c_2)\bar{v}}{2c_1c_2} = TE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right).
\end{aligned}
$$

Therefore, choosing $\gamma \in \{CC, DD\}$ with $\delta = \frac{c_2}{c_1}$ generates strictly more expected total effort to the designer than choosing $\gamma = DC$ with any $\delta > 0$. Recall that $TE(DC, \delta; c_1, c_2) = TE(CD, 1/\delta; c_2, c_1)$. This immediately implies that choosing $\gamma \in \{CC, DD\}$ with $\delta = \frac{c_2}{c_1}$ generates strictly more expected total effort for the designer than choosing $\gamma = CD$ with any $\delta > 0$.

Next, we prove part (ii). It is useful to prove an intermediate result.

Lemma A3 *Fix $q \in (1/2, 1]$. $WE(DC, \delta; c_1, c_2)$ is maximized at $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{c_2}{\hat{\mu}(q)c_1}$.*

Proof. Fix $u \in (0,1)$ and $z \in \mathbb{R}_{++}$. First, for $d \in (0,1)$, we have that

$$\frac{\partial \mathcal{W}_1(u,z,d;c_1,c_2)}{\partial d} = u^2 z\left[(3-u)c_1 + (3-2u)c_2\right] + (2c_1+c_2) + 2\left(c_2u^3z + c_2\right)(1-d) > 0.$$

Therefore, $\mathcal{W}_1(u,z,d;c_1,c_2)$ is increasing in d for $d \in (0,1)$.

Next, we show that $\mathcal{W}_2(u,z,d;c_1,c_2)$, with $d \in [1,1/\mu]$, is maximized at $d=1$ or $d=1/u$. Simple algebra would verify that

$$\frac{\partial \mathcal{W}_2(u,z,d;c_1,c_2)}{\partial d} = \frac{\left[zu^2\mathcal{W}_4(u,d;c_1,c_2)-1\right](3c_1d+2c_2d-2c_1)}{d^3},$$

where $\mathcal{W}_4(u,d;c_1,c_2) := \frac{3(c_1+c_2)-u(c_1+2c_2d)}{3c_1d+2c_2d-2c_1}d^3$. Note that

$$\frac{\partial \mathcal{W}_4(u,d;c_1,c_2)}{\partial d} = \frac{6d^2\mathcal{W}_5(u,d;c_1,c_2)}{\left[c_1(3d-2)+2c_2d\right]^2},$$

where $\mathcal{W}_5(u,d;c_1,c_2) := -c_2u(3c_1+2c_2)d^2+\left[c_1^2(3-u)+c_1c_2(2u+5)+2c_2^2\right]d+c_1\left[c_1u-3(c_1+c_2)\right]$. Note that $\mathcal{W}_5(u,d;c_1,c_2)$ is concave in d, which implies that

$$\mathcal{W}_5(u,d;c_1,c_2) \geq \min\left\{\mathcal{W}_5(u,1;c_1,c_2), \mathcal{W}_5(u,1/u;c_1,c_2)\right\}, \text{ for } d \in [1,1/\mu];$$

together with $\mathcal{W}_5(u,1;c_1,c_2) = 2c_2(c_1+c_2) - c_2u(c_1+2c_2) > 0$ and $\mathcal{W}_5(u,1/u;c_1,c_2) = \frac{c_1(c_1(3-u)(1-u)+c_2(2-u))}{u} > 0$, we can conclude that $\mathcal{W}_5(u,d;c_1,c_2) > 0$. As a result, $\frac{\partial \mathcal{W}_4(u,d;c_1,c_2)}{\partial d} > 0$ and thus $\mathcal{W}_4(u,d;c_1,c_2)$ is increasing in d for $d \in [1,1/u]$, which in turn implies that

$$\frac{\partial \mathcal{W}_2(u,z,d;c_1,c_2)}{\partial d} \gtreqless 0 \Leftrightarrow zu^2\mathcal{W}_4(u,d;c_1,c_2) \gtreqless 1.$$

Therefore, $\mathcal{W}_2(u,z,d;c_1,c_2)$ is either monotonic or U-shaped in $d \in [1,1/u]$. This implies that $\mathcal{W}_2(u,z,d;c_1,c_2)$ is maximized at $d=1$ or $d=1/u$.

Finally, for $d>1$, we have that

$$\frac{\partial \mathcal{W}_3(d;c_1,c_2)}{\partial d} = -\frac{3c_1d-2c_1+2c_2d}{d^3} < 0,$$

which implies that $\mathcal{W}_3(d;c_1,c_2)$ is decreasing in d for $d>1$.

In summary, (i) $\mathcal{W}_1(u,z,d;c_1,c_2)$ is increasing in d for $d \in (0,1)$; (ii) $\mathcal{W}_2(u,z,d;c_1,c_2)$ is maximized at $d=1$ or $d=1/u$; and (iii) $\mathcal{W}_3(d;c_1,c_2)$ is decreasing in d for $d>1$. All together, these facts imply that $WE(DC,\delta;c_1,c_2)$ is maximized at $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{c_2}{\hat{\mu}(q)c_1}$, which concludes the proof. ■

For $\gamma \in \{CC, DD\}$, we have that

$$\frac{\partial WE(CC,\delta;c_1,c_2)}{\partial\delta} = \frac{\partial WE(DD,\delta;c_1,c_2)}{\partial\delta} = \begin{cases} \frac{(3c_2-2c_1\delta+2c_1)\bar{v}}{6c_2^2} > 0, & \text{if } \delta < \frac{c_2}{c_1}; \\ -\frac{(3c_1\delta-2c_2+2c_2\delta)\bar{v}}{6c_1^2\delta^3} < 0, & \text{if } \delta \geq \frac{c_2}{c_1}. \end{cases}$$

Therefore, $WE(CC,\delta;c_1,c_2)$ and $WE(DD,\delta;c_1,c_2)$ are both maximized at $\delta = \frac{c_2}{c_1}$. The maximum expected winner's effort is $\frac{(c_1+c_2)\bar{v}}{3c_1c_2}$.

Further, fixing $q \in (1/2, 1]$, it follows from Lemma A3 that $WE(DC,\delta;c_1,c_2)$ is maximized at $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{c_2}{\hat{\mu}(q)c_1}$. Carrying out the algebra, we can obtain that

$$\begin{aligned} WE\left(DC,\frac{c_2}{c_1};c_1,c_2\right) &= \frac{(c_1+c_2)\left\{2\bar{v} - \left[2-\hat{\mu}(q)\right]\left[1-\hat{\mu}(q)\right]\hat{\mu}(q)\left[\hat{v}_H(q)-\hat{v}_L(q)\right]\right\}}{6c_1c_2} \\ &< \frac{(c_1+c_2)\bar{v}}{3c_1c_2} = WE\left(CC,\frac{c_2}{c_1};c_1,c_2\right), \end{aligned}$$

and

$$WE\left(DC,\frac{c_2}{\hat{\mu}(q)c_1};c_1,c_2\right) = \frac{\hat{\mu}(q)\hat{v}_H(q)\left\{2c_2 + c_1\left[3-\hat{\mu}(q)\right]\right\}}{6c_1c_2}.$$

Further, recall that $WE(CD,\delta;c_1,c_2) = WE(DC,1/\delta;c_2,c_1)$; together with the above analysis, we can conclude that $WE(CD,\delta;c_1,c_2)$ is maximized at $\delta = \frac{c_2}{c_1}$ or $\delta = \frac{\hat{\mu}(q)c_2}{c_1}$. Moreover, we have that

$$WE\left(CD,\frac{c_2}{c_1};c_1,c_2\right) = WE\left(DC,\frac{c_1}{c_2};c_2,c_1\right) = WE\left(DC,\frac{c_2}{c_1};c_1,c_2\right) < WE\left(CC,\frac{c_2}{c_1};c_1,c_2\right),$$

and

$$\begin{aligned} WE\left(CD,\frac{\hat{\mu}(q)c_2}{c_1};c_1,c_2\right) &= \frac{\hat{\mu}(q)\hat{v}_H(q)\left\{2c_1 + c_2\left[3-\hat{\mu}(q)\right]\right\}}{6c_1c_2} \\ &< \frac{\hat{\mu}(q)\hat{v}_H(q)\left\{2c_2 + c_1\left[3-\hat{\mu}(q)\right]\right\}}{6c_1c_2} = WE\left(DC,\frac{c_2}{\hat{\mu}(q)c_1};c_1,c_2\right), \end{aligned}$$

where the strict inequality follows from $c_1 \geq c_2$ and $3-\hat{\mu}(q) > 2$. As a result, $\gamma = CD$ would not arise in the optimum.

In summary, fixing $q \in (1/2, 1]$, the expected winner's effort from the contest is maximized by either $(\gamma,\delta) = \left(CC \text{ or } DD, \frac{c_2}{c_1}\right)$ or $(\gamma,\delta) = \left(DC, \frac{c_2}{\hat{\mu}(q)c_1}\right)$. Carrying out the algebra, we have that

$$WE\left(CC, \frac{c_2}{c_1}; c_1, c_2\right) - WE\left(DC, \frac{c_2}{\hat{\mu}(q)c_1}; c_1, c_2\right) = \frac{\big[1-\hat{\mu}(q)\big] \times \big[2(c_1+c_2)\hat{v}_L(q) - c_1\hat{\mu}(q)\hat{v}_H(q)\big]}{6c_1c_2}.$$

It can be verified that $WE\big(CC, \frac{c_2}{c_1}; c_1, c_2\big) > WE\big(DC, \frac{c_2}{\hat{\mu}(q)c_1}; c_1, c_2\big)$ is equivalent to $\hat{\mu}(q)\hat{v}_H(q) < \big(2\frac{c_2}{c_1}+2\big)\hat{v}_L(q)$, which concludes the proof. ■

Proof of Proposition 3

Proof. To proceed, we derive how the designer's deviation in disclosure policy changes a players' bidding strategy. Due to symmetry, it is without loss to focus on player 2. Further, when $\gamma(2) = D$, either player 2 cannot detect and react to the designer's deviation, or he would complain for sure due to loss of information, rendering the deviation infeasible. Therefore, it suffices to consider the case in which $\gamma(2) = C$.

Let $\bar{b}_{1s}(\gamma, \delta)$ denote player 1's highest equilibrium bid under (γ, δ) and the signal realization s for $\gamma \in \{DC, DD\}$. Similarly, let $\bar{b}_1(\gamma, \delta)$ denote player 1's highest equilibrium bid under (γ, δ) for $\gamma \in \{CC, CD\}$. The following result ensues.

Lemma A4 ***(Player's Bidding Strategy upon Detecting Designer's Deviation)*** *Fix an announced policy (γ, δ) and the designer's deviation $\gamma' \neq \gamma$. Player 2's bidding strategy is described as follows.*

(i) If $\gamma = CC$ and $\gamma' \in \{CD, DD\}$, player 2 bids 0 when receiving a low signal and bids $\bar{b}_1(CC, \delta)/\delta$ when receiving a high signal.

(ii) If $\gamma = CC$ and $\gamma' = DC$, player 2 follows his equilibrium bidding strategy under (γ, δ) since he is not aware of the deviation.

(iii) If $\gamma = DC$ and $\gamma' = DD$, player 2 bids $\bar{b}_{1s}(DC, \delta)/\delta$ when receiving signal $s \in \{L, H\}$ and wins the contest with certainty.

Proof of Lemma A4

Proof. It is straightforward to verify that the strategies described in the lemma are player 2's best responses to player 1's equilibrium bidding strategy under (γ, δ) following the designer's deviations. ■

For the proof, we assume $c = 1$ without loss of generality. Part (i) follows immediately from Lemma A4 and it remains to prove part (ii).

For part (ii), first note that the contest $(\gamma, \delta) = (DD, 1)$ is credible. Therefore, in the case in which $(DD, 1)$ or $(CC, 1)$ is optimal absent credibility concern—i.e., when $\hat{\mu}(q)\hat{v}_H(q) \leq$

$4\hat{v}_L(q)$—$(DD, 1)$ is still optimal in the presence of credibility concern. Further, note that $WE(CC, 1) = WE(DD, 1)$. As long as $(\gamma, \delta) = (DD, 1)$ emerges as an optimal credible contest, $(CC, 1)$ is also optimal, provided that it is credible. By Lemma A4, we can verify that $(CC, 1)$ is credible if and only if $\hat{\mu}(q) \leq 1/3$.

It remains to consider the case in which $\hat{\mu}(q)\hat{v}_H(q) > 4\hat{v}_L(q)$, or equivalently, $WE(DC, 1/[\hat{\mu}(q)]) > WE(DD, 1)$. By Lemma A4, fixing δ, the designer's deviation from DC to DD generates the following expected winner's effort:

$$WE(DC \to DD, \delta) := \begin{cases} [1 - \hat{\mu}(q)]\hat{v}_L(q) + [\hat{\mu}(q)]^2\hat{v}_H(q), & \text{if } \delta < 1, \\ \frac{1-\hat{\mu}(q)\delta}{\delta}\hat{v}_L(q) + [\hat{\mu}(q)]^2\hat{v}_H(q), & \text{if } 1 \leq \delta \leq \frac{1}{\hat{\mu}(q)}, \\ \frac{\hat{\mu}(q)\hat{v}_H(q)}{\delta}, & \text{if } \delta > \frac{1}{\hat{\mu}(q)}. \end{cases}$$

Fixing $\delta = 1/[\hat{\mu}(q)]$, simple algebra would verify that $WE(DC \to DD, \delta) \leq WE(DC, \delta)$ if and only if $\hat{\mu}(q) \leq 5/7$. Therefore, the optimal credible contest is $(DC, 1/[\hat{\mu}(q)])$ or $(CD, \hat{\mu}(q))$ if $\hat{\mu}(q) \leq 5/7$.

We now turn to the case of $\hat{\mu}(q) > 5/7$. Carrying out the algebra, we can show that $WE(DC \to DD, \delta) > WE(DC, \delta)$ for all $\delta \leq 1/[\hat{\mu}(q)]$. Put differently, any contest (DC, δ) with $\delta \leq 1/[\hat{\mu}(q)]$ is not credible. For $\delta > 1/[\hat{\mu}(q)]$, it holds that

$$WE(DC, \delta) - WE(DC \to DD, \delta) = \frac{[5 - 6\hat{\mu}(q)]\delta - 1}{6\delta^2}\hat{v}_H(q).$$

Suppose $\hat{\mu}(q) > 5/6$. It can be verified that $WE(DC, \delta) - WE(DC \to DD, \delta) < 0$ and thus (DC, δ) is not credible for all $\delta > 1/[\hat{\mu}(q)]$, from which we can conclude that $(DD, 1)$ is the optimal credible contest.

Next, suppose $5/7 < \hat{\mu}(q) \leq 5/6$. It follows from the above equation that (DC, δ) is credible if and only if $\delta \geq 1/[5-6\hat{\mu}(q)]$. Recall from the proof of Lemma A3 that $WE(DC, \delta)$ decreases with δ for $\delta > 1/[\hat{\mu}(q)]$. Further, $1/[5-6\hat{\mu}(q)] > 1/[\hat{\mu}(q)]$ for $\hat{\mu}(q) > 5/7$. Therefore, $\delta = 1/[5 - 6\hat{\mu}(q)]$ maximizes the expected winner's effort among all credible contests with $\gamma = DC$. Moreover, simple algebra would verify that $WE(DC, 1/[5-6\hat{\mu}(q)]) > WE(DD, 1)$ is equivalent to

$$\frac{13\hat{\mu}(q) - 18[\hat{\mu}(q)]^2}{2[1 - \hat{\mu}(q)]} > \frac{\hat{v}_L(q)}{\hat{v}_H(q)}.$$

Therefore, both $(\gamma^*_{WE}, \delta^*_{WE}) = \big(CD, 5 - 6\hat{\mu}(q)\big)$ and $(\gamma^*_{WE}, \delta^*_{WE}) = \big(DC, 1/[5 - 6\hat{\mu}(q)]\big)$ are an optimal credible contest scheme if the above inequality holds and $(DD, 1)$ is optimal otherwise. ■

Proof of Proposition 4

Proof. For the proof, we assume $c = 1$ without loss of generality. From the equilibrium characterization results, we can obtain the following:

$$ME(CC, 1) = \frac{2}{3}\bar{v}, \text{ and } ME\left(CD, \hat{\mu}(q)\right) = \frac{\left\{3 + 3[1 - \hat{\mu}(q)] + [\hat{\mu}(q)]^2\right\} \hat{\mu}(q)\hat{v}_H(q)}{6}.$$

It can be verified that $ME(CC, 1) > ME\left(CD, \hat{\mu}(q)\right)$ is equivalent to

$$\frac{\hat{v}_L(q)}{\hat{v}_H(q)} > \frac{\hat{\mu}(q)\left[2 - \hat{\mu}(q)\right]}{4},$$

which concludes the proof. ■

Proof of Lemma 1

Proof. See Appendix C. ■

Proof of Proposition 5

Proof. For the proof, we assume $c = 1$ without loss of generality. It is useful to prove an intermediate result.

Lemma A5 *Suppose that $\gamma = DC$. Fix an arbitrary tuple $(v_H^\pi, v_L^\pi, \mu^\pi)$ that satisfies* (3) *and let the designer set the scoring bias $\delta > 0$. Then the expected winner's effort from the contest is maximized at $\delta = 1$ or $\delta = \frac{1}{\mu^\pi}$.*

Proof. The proof closely follows that of Lemma A3 and is omitted for brevity. ■

Following the same steps in the proof of Proposition 2, we can show that for an arbitrary tuple $(v_H^\pi, v_L^\pi, \mu^\pi)$ that satisfies (3), the expected winner's effort from the contest is maximized by $(\delta, \gamma) = (1, CC)$, $(\delta, \gamma) = (1, DD)$, or $(\delta, \gamma) = \left(\frac{1}{\mu^\pi}, DC\right)$. The first two contest schemes generate an expected winner's effort of $\frac{2}{3}\bar{v}$, while the third one generates an expected winner's effort of $\frac{\mu^\pi v_H^\pi(5-\mu^\pi)}{6}$. The optimization problem under $\gamma = DC$ is

$$\max_{v_L^\pi, v_H^\pi, \mu^\pi} \frac{\mu^\pi v_H^\pi(5 - \mu^\pi)}{6} \; s.t. \; (3).$$

It can be verified that for an arbitrary μ^π, suppose that $\sum_{\ell=k+1}^K \mu_\ell < \mu^\pi \leq \sum_{\ell=k}^K \mu_\ell$ for some k, then the largest v_H^π satisfying (3) is given by

$$\frac{[\mu^\pi - \Pr(v > v_k)]v_k + \Pr(v > v_k)\mathbb{E}[v|v > v_k]}{\mu^\pi}.$$

So the optimization problem becomes

$$\max_{\mu^\pi} W(\mu^\pi) := \frac{\left\{[\mu^\pi - \Pr(v > v_k)]v_k + \Pr(v > v_k)\mathbb{E}[v|v > v_k]\right\}(5 - \mu^\pi)}{6},$$

where k satisfies $\sum_{\ell=k+1}^{K} \mu_\ell < \mu^\pi \leq \sum_{\ell=k}^{K} \mu_\ell$. For a fixed k, $W(\mu^\pi)$ is quadratic in μ^π and the axis of symmetry is

$$\mu^*(k) := 3 - \frac{\sum_{\ell=1}^{k} \mu_\ell}{2} - \frac{\sum_{\ell=k+1}^{K} \mu_\ell v_\ell}{2v_k},$$

which increases with k. Note that $\sum_{\ell=k+1}^{K} \mu_\ell$ and $\sum_{\ell=k}^{K} \mu_\ell$ both decrease with k. Let $k^* := \min\left\{k : \mu^*(k) \geq \sum_{\ell=k+1}^{K} \mu_\ell\right\}$. Then for all $k > k^*$, $\mu^*(k) \geq \mu^*(k^*) \geq \sum_{\ell=k^*+1}^{K} \mu_\ell \geq \sum_{\ell=k}^{K} \mu_\ell$. Therefore, $W(\mu^\pi)$ increases with $\mu^\pi \leq \sum_{\ell=k^*+1}^{K} \mu_\ell$. Meanwhile, for all $k < k^*$, $\mu^*(k) < \sum_{\ell=k+1}^{K} \mu_\ell$. So $W(\mu^\pi)$ decreases with $\mu^\pi > \sum_{\ell=k^*}^{K} \mu_\ell$. Finally, it is easy to see that for $\sum_{\ell=k^*+1}^{K} \mu_\ell < \mu^\pi \leq \sum_{\ell=k^*}^{K} \mu_\ell$, $W(\mu^\pi)$ increases with $\mu^\pi \leq \min\left\{\sum_{\ell=k^*}^{K} \mu_\ell, \mu^*(k^*)\right\}$ and decreases otherwise.

To sum up, $W(\mu^\pi)$ is maximized at $\mu^\pi = \min\left\{\sum_{\ell=k^*}^{K} \mu_\ell, \mu^*(k^*)\right\}$. In the case of $\mu^*(1) \geq 1$, which is equivalent to $\bar{v}/v_1 \leq 4$, $W(\mu^\pi)$ is maximized at $\mu^\pi = 1$. The maximized value is $\frac{2}{3}\bar{v}$, so $(\delta, \gamma) = (1, CC)$ and $(\delta, \gamma) = (1, DD)$ are optimal. In the case of $\mu^*(1) < 1$, the maximized value $W(\mu^\pi) > W(1) = \frac{2}{3}\bar{v}$, so $\gamma = DC$ is optimal. Finally, it is straightforward to verify the desired distribution of posterior expectations can be induced by the signal structure described in the proposition. ■

Appendix C General Information Structure

This appendix concerns the case where the designer can freely choose the information structure. We show that binary signals are optimal for maximizing the expected winner's effort. We assume $c = 1$ without loss of generality and focus on the $\gamma = DC$ case.

Fix $\delta \in (0, \infty)$. We first characterize the equilibrium under an arbitrary information structure $\pi(s|v)$. Suppose that the corresponding posterior belief is given by

$$\left\langle (v_1, \dots, v_N), (\mu_1, \dots, \mu_N) \right\rangle,$$

where $v_n := \mathbb{E}(v|s_n)$ and $\mu_n := \Pr(s_n) > 0$. We order the signals such that the following is satisfied,

$$v_L \le v_1 < \cdots < v_N \le v_H, \ \sum_{n=1}^{N} \mu_n = 1, \text{ and } \sum_{n=1}^{N} \mu_n v_n = \bar{v}.$$

Proposition A3 *Given the posterior* $\langle (v_1, \dots, v_N), (\mu_1, \dots, \mu_N) \rangle$ *and* $\gamma = DC$*, the equilibrium of the contest is described as follows.*

(i) Suppose that $\delta < 1$*. Contestant 1, upon receiving signal* s_n*, mixes uniformly over the interval* $\left[\delta \sum_{k=1}^{n-1} \mu_k v_k, \delta \sum_{k=1}^{n} \mu_k v_k\right]$*. Contestant 2 exerts 0 effort with probability* $1 - \delta$*, and mixes over the interval* $\left[\sum_{k=1}^{n-1} \mu_k v_k, \sum_{k=1}^{n} \mu_k v_k\right]$ *with density* $\frac{\delta}{v_n}$ *for* $n \in \{1, \dots, N\}$*.*

(ii) Suppose that $\delta \ge 1$*. Further, suppose that* $\frac{1}{\sum_{k=n_0}^{N} \mu_k} \le \delta < \frac{1}{\sum_{k=n_0+1}^{N} \mu_k}$ *for some* $n_0 \in \{1, \dots, N\}$*. Contestant 1, upon receiving a signal from* $\{s_1, \dots, s_{n_0-1}\}$*, exerts 0 effort for sure; upon receiving signal* s_{n_0}*, he exerts 0 effort with probability* $1 - \frac{1}{\delta \mu_{n_0}} \left(1 - \delta \sum_{k=n_0+1}^{N} \mu_k\right)$*, and mixes over the interval* $\left[0, v_{n_0} \left(1 - \delta \sum_{k=n_0+1}^{N} \mu_k\right)\right]$ *with density* $\frac{1}{\delta \mu_{n_0} v_{n_0}}$*; upon receiving signal* $s_n \in \{s_{n_0+1}, \dots, s_N\}$*, he mixes uniformly over the interval*
$\left[v_{n_0} \left(1 - \delta \sum_{k=n_0+1}^{N} \mu_k\right) + \delta \sum_{k=n_0+1}^{n-1} \mu_k v_k, v_{n_0} \left(1 - \delta \sum_{k=n_0+1}^{N} \mu_k\right) + \delta \sum_{k=n_0+1}^{n} \mu_k v_k\right]$.
Contestant 2 mixes over the interval $\left[0, \frac{v_{n_0}}{\delta} \left(1 - \delta \sum_{k=n_0+1}^{N} \mu_k\right)\right]$ *with density* $\frac{\delta}{v_{n_0}}$ *and the interval*
$\left[\frac{v_{n_0}}{\delta} \left(1 - \delta \sum_{k=n_0+1}^{N} \mu_k\right) + \sum_{k=n_0+1}^{n-1} \mu_k v_k, \frac{v_{n_0}}{\delta} \left(1 - \delta \sum_{k=n_0+1}^{N} \mu_k\right) + \sum_{k=n_0+1}^{n} \mu_k v_k\right]$ *with density* $\frac{\delta}{v_n}$ *for* $n \in \{n_0 + 1, \dots, N\}$*.*

Next, we show that a binary signal space is optimal. In particular, suppose that $N \ge 3$, we show that there exists a binary state information structure that performs weakly better than $\pi(s|v)$.

Case (i): $\boldsymbol{\delta \leq \frac{1}{\mu_{N-1}+\mu_N}}$. In this case, contestant 1 mixes uniformly if he receives signal s_{N-1} or s_N. We denote by $\tilde{x}$ the left endpoint of the interval contestant 1 mixes on when the signal is s_{N-1}. That is, contestant 1 mixes over $[\delta\tilde{x}, \delta(\tilde{x}+\mu_{N-1}v_{N-1})]$ if he receives signal s_{N-1}, and he mixes over $[\delta(\tilde{x}+\mu_{N-1}v_{N-1}), \delta(\tilde{x}+\mu_{N-1}v_{N-1}+\mu_N v_N)]$ if he receives signal s_N.

The expected winner's effort in equilibrium can be calculated as follows.

$$\begin{aligned}
WE(\pi) =& \Pr(x_1 < \delta\tilde{x}, x_2 < \tilde{x})\mathbb{E}(x_1\mathbb{1}_{\{x_1>\delta x_2\}} + x_2\mathbb{1}_{\{x_1<\delta x_2\}}|x_1 < \delta\tilde{x}, x_2 < \tilde{x}) \\
&+ \Pr(x_1 \geq \delta\tilde{x}, x_2 \geq \tilde{x})\mathbb{E}(x_1\mathbb{1}_{\{x_1>\delta x_2\}} + x_2\mathbb{1}_{\{x_1<\delta x_2\}}|x_1 \geq \delta\tilde{x}, x_2 \geq \tilde{x}) \\
&+ \Pr(x_1 < \delta\tilde{x}, x_2 \geq \tilde{x})\mathbb{E}(x_2|x_2 \geq \tilde{x}) + \Pr(x_1 \geq \delta\tilde{x})\mathbb{E}(x_1|x_1 \geq \delta\tilde{x}).
\end{aligned}$$

Consider an alternative information structure $\hat{\pi}(s|v)$ with one less signal realization. In particular, let the posterior induced by $\hat{\pi}(s|v)$ be $\big\langle(v_1, \ldots, v_{N-2}, \hat{v}), (\mu_1, \ldots, \mu_{N-2}, \hat{\mu})\big\rangle$, where $\hat{v} := \frac{\mu_{N-1}v_{N-1}+\mu_N v_N}{\mu_{N-1}+\mu_N}$ and $\hat{\mu} := \mu_{N-1}+\mu_N$. Then the equilibrium strategy of contestant 1 is the same under the two information structures if he receives $s_n \in \{s_1, \ldots, s_{N-2}\}$, or equivalently, $x_1 < \delta\tilde{x}$. Contestant 2's equilibrium effort distribution is also the same under the two information structures conditional on $x_2 < \tilde{x}$. Moreover,

$$\Pr(x_1 < \delta\tilde{x}) = \widehat{\Pr}(x_1 < \delta\tilde{x}) \text{ and } \Pr(x_2 < \tilde{x}) = \widehat{\Pr}(x_2 < \tilde{x}),$$

where $\widehat{\Pr}$ represents probability under information structure $\hat{\pi}(s|v)$. Therefore, to compare the two information structures, it suffices to focus on the case where either $x_1 \geq \delta\tilde{x}$ or $x_2 \geq \tilde{x}$ (or both).

To show that $WE(\pi) < WE(\hat{\pi})$, it suffices to prove that

$$\mathbb{E}(x_1\mathbb{1}_{\{x_1>\delta x_2\}} + x_2\mathbb{1}_{\{x_1<\delta x_2\}}|x_1 \geq \delta\tilde{x}, x_2 \geq \tilde{x}) < \widehat{\mathbb{E}}(x_1\mathbb{1}_{\{x_1>\delta x_2\}} + x_2\mathbb{1}_{\{x_1<\delta x_2\}}|x_1 \geq \delta\tilde{x}, x_2 \geq \tilde{x}).$$

In fact, we have that

$$\begin{aligned}
&\mathbb{E}(x_1\mathbb{1}_{\{x_1>\delta x_2\}} + x_2\mathbb{1}_{\{x_1<\delta x_2\}}|x_1 \geq \delta\tilde{x}, x_2 \geq \tilde{x}) \\
=&\frac{1}{\delta v_{N-1}^2(\mu_{N-1}+\mu_N)^2}\Bigg[\int_{\delta\tilde{x}}^{\delta(\tilde{x}+\mu_{N-1}v_{N-1})} dx_1 \int_{\tilde{x}}^{x_1/\delta} x_1 dx_2 \\
&\qquad + \int_{\delta\tilde{x}}^{\delta(\tilde{x}+\mu_{N-1}v_{N-1})} dx_1 \int_{x_1/\delta}^{\tilde{x}+\mu_{N-1}v_{N-1}} x_2 dx_2\Bigg] \\
&+ \frac{1}{\delta v_N^2(\mu_{N-1}+\mu_N)^2}\Bigg[\int_{\delta(\tilde{x}+\mu_{N-1}v_{N-1})}^{\delta(\tilde{x}+\mu_{N-1}v_{N-1}+\mu_N v_N)} dx_1 \int_{\tilde{x}+\mu_{N-1}v_{N-1}}^{x_1/\delta} x_1 dx_2 \\
&\qquad + \int_{\delta(\tilde{x}+\mu_{N-1}v_{N-1})}^{\delta(\tilde{x}+\mu_{N-1}v_{N-1}+\mu_N v_N)} dx_1 \int_{x_1/\delta}^{\tilde{x}+\mu_{N-1}v_{N-1}+\mu_N v_N} x_2 dx_2\Bigg]
\end{aligned}$$

$$+\frac{\mu_{N-1}\mu_N}{(\mu_{N-1}+\mu_N)^2}(1+\delta)\frac{2\tilde{x}+2\mu_{N-1}v_{N-1}+\mu_N v_N}{2}$$
$$=\frac{(1+\delta)[(2\mu_{N-1}^3+6\mu_{N-1}^2\mu_N+3\mu_{N-1}\mu_N^2)v_{N-1}+(3\mu_{N-1}\mu_N^2+2\mu_N^3)v_N+3c_2(\mu_{N-1}+\mu_N)^2\tilde{x}]}{6c_2(\mu_{N-1}+\mu_N)^2}$$

and

$$\begin{aligned}
&\widehat{\mathbb{E}}(x_1\mathbb{1}_{\{x_1>\delta x_2\}}+x_2\mathbb{1}_{\{x_1<\delta x_2\}}|x_1\geq\delta\tilde{x},x_2\geq\tilde{x})\\
=&\frac{1}{\delta(\mu_{N-1}v_{N-1}+\mu_N v_N)^2}\Bigg[\int_{\delta\tilde{x}}^{\delta(\tilde{x}+\mu_{N-1}v_{N-1}+\mu_N v_N)}dx_1\int_{\tilde{x}}^{x_1/\delta}x_1dx_2\\
&\qquad+\int_{\delta\tilde{x}}^{\delta(\tilde{x}+\mu_{N-1}v_{N-1}+\mu_N v_N)}dx_1\int_{x_1/\delta}^{\tilde{x}+\mu_{N-1}v_{N-1}+\mu_N v_N}x_2dx_2\Bigg]\\
=&\frac{(1+\delta)(2\mu_{N-1}v_{N-1}+2\mu_N v_N+3\tilde{x})}{6}.
\end{aligned}$$

Simple algebra yields that

$$\begin{aligned}
&\widehat{\mathbb{E}}(x_1\mathbb{1}_{\{x_1>\delta x_2\}}+x_2\mathbb{1}_{\{x_1<\delta x_2\}}|x_1\geq\delta\tilde{x},x_2\geq\tilde{x})-\mathbb{E}(x_1\mathbb{1}_{\{x_1>\delta x_2\}}+x_2\mathbb{1}_{\{x_1<\delta x_2\}}|x_1\geq\delta\tilde{x},x_2\geq\tilde{x})\\
=&\frac{(1+\delta)\mu_{N-1}\mu_N(2\mu_{N-1}+\mu_N)(v_N-v_{N-1})}{6(\mu_{N-1}+\mu_N)^2}>0.
\end{aligned}$$

By applying the procedure (pooling the highest two signals as long as $\frac{1}{\mu_{N-1}+\mu_N}\geq\delta$) repeatedly, we can either (i) reduce the number of signal realizations to be less than or equal to 2, or (ii) proceed to the next case.

Case (ii): $\boldsymbol{\frac{1}{\mu_{N-1}+\mu_N}<\delta<\frac{1}{\mu_N}}$. In this case, only type-s_{N-1} and type-s_N contestant 1 is active in equilibrium. The expected winner's effort is calculated as follows.

$$WE(\pi)=\frac{(1+\delta)\left[\delta^2\mu_N^2(3-\delta\mu_N)(v_N-v_{N-1})+2v_{N-1}\right]}{6\delta^2}.$$

It is worth pointing out that $WE(\pi)$ is independent of μ_{N-1} and is increasing in $\mu_N\in\left(0,\frac{1}{\delta}\right)$.

Consider an alternative information structure $\pi'(s|v)$ generated by the following change to $\pi(s|v)$: when s_{N-1} is drawn according to $\pi(s|v)$, $\pi'(s|v)$ changes the signal to s_1 with probability $\frac{\mu_{N-1}-\left(\frac{1}{\delta}-\mu_N\right)}{\mu_{N-1}}$ and sends s_{N-1} with the complementary probability; when other signals are drawn according to $\pi(s|v)$, $\pi'(s|v)$ sends the same signal. Specifically,

$$\begin{aligned}
\pi'(s_1|v) &= \pi(s_1|v) + \pi(s_{N-1}|v)\frac{\mu_{N-1} - \left(\frac{1}{\delta} - \mu_N\right)}{\mu_{N-1}}, \\
\pi'(s_{N-1}|v) &= \pi(s_{N-1}|v)\frac{\frac{1}{\delta} - \mu_N}{\mu_{N-1}}, \text{ and} \\
\pi'(s_k|v) &= \pi(s_k|v), \text{ for } k \neq 1, N-1.
\end{aligned}$$

Since $WE(\pi)$ is independent of μ_{N-1}, and $\pi'(s|v)$ simply reduces this posterior without changing v_N, v_{N-1}, or μ_N, we have that $WE(\pi) = WE(\pi')$. But by construction, $\frac{1}{\mu'_{N-1}+\mu'_N} = \delta$, which brings us back to Case (i), implying that the highest two signals should be pooled together. Since we have a finite number of signals, eventually this procedure will reduce the number of signals to two.

Case (iii): $\boldsymbol{\delta \geq \frac{1}{\mu_N}}$. In this case, only type-s_N contestant 1 is active in equilibrium. The information structure $\pi(s|v)$ is equivalent to a binary state information structure with the following posterior,

$$\hat{v}_1 = \frac{\sum_{k=1}^{N-1} \mu_k v_k}{\sum_{k=1}^{N-1} \mu_k}, \hat{v}_2 = v_N, \text{ and } \hat{\mu}_1 = \sum_{k=1}^{N-1} \mu_k, \tilde{\mu}_2 = \mu_N.$$

In conclusion, $\pi(s|v)$ is weakly outperformed by a binary state information structure.